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EDEXCEL INTERNATIONAL GCSE (9–1)

MATHEMATICS A

Student Book 1

David Turner, Ian Potts

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MATHEMATICS A

Student Book 1

David Turner,
Ian Potts

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There are links to relevant websites in this book. In order to ensure that the links are up to date and that the links work we have made the links available on our website at www.pearsonhotlinks.co.uk. Search for ISBN 978 0 435 18144 4.

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UNIT 4

UNIT 5

ABOUT THIS BOOK

This two-book series is written for students following the Edexcel International GCSE (9-1) Maths A Higher Tier specification. There is a Student Book for each year of the course.

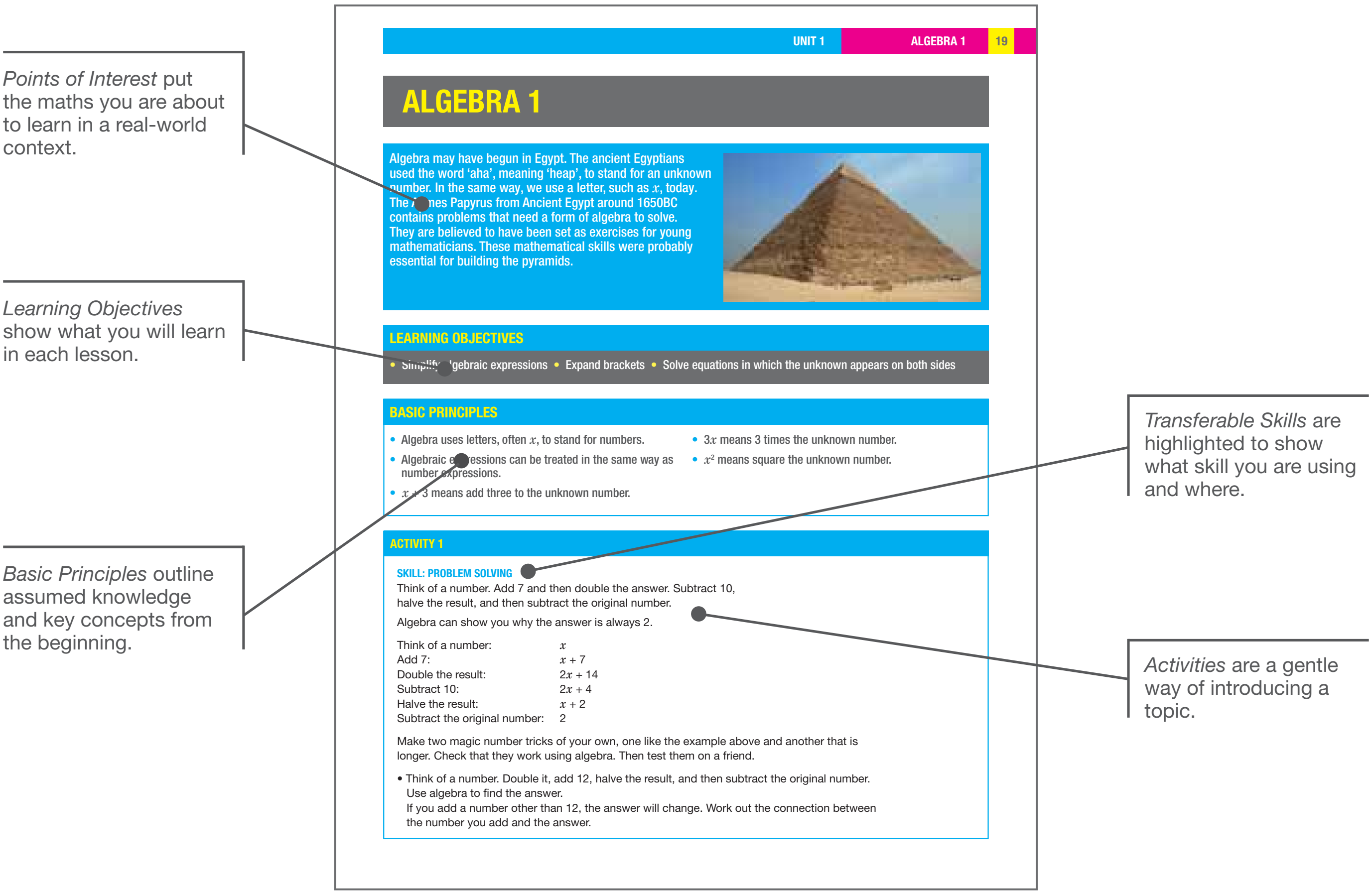
The course has been structured so that these two books can be used in order, both in the classroom and for independent learning.

Each book contains five units of work. Each unit contains five sections in the topic areas: *Number, Algebra, Graphs*, *Shape and Space* and *Handling Data*.

In each unit, there are concise explanations and worked examples, plus numerous exercises that will help you build up confidence.

Parallel exercises, non-starred and starred, are provided, to bring together basic principles before being challenged with more difficult questions. These are supported by parallel revision exercises at the end of each chapter.

Challenges, which provide questions applying the basic principles in unusual situations, feature at the back of the book along with *Fact Finders* which allow you to practise comprehension of real data.

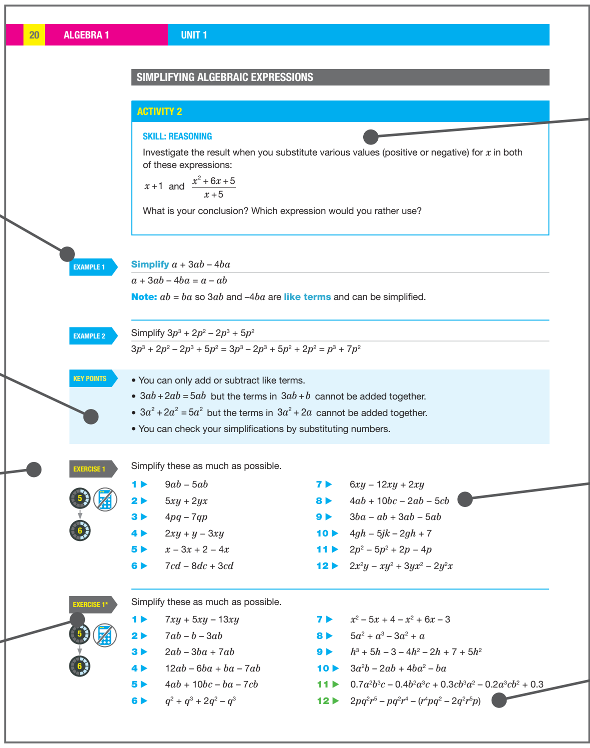


Examples provide a clear, instructional framework.

Key Points boxes summarise the essentials.

Progression icons show the level of difficulty according to the Pearson International GCSE Maths Progression Scale.

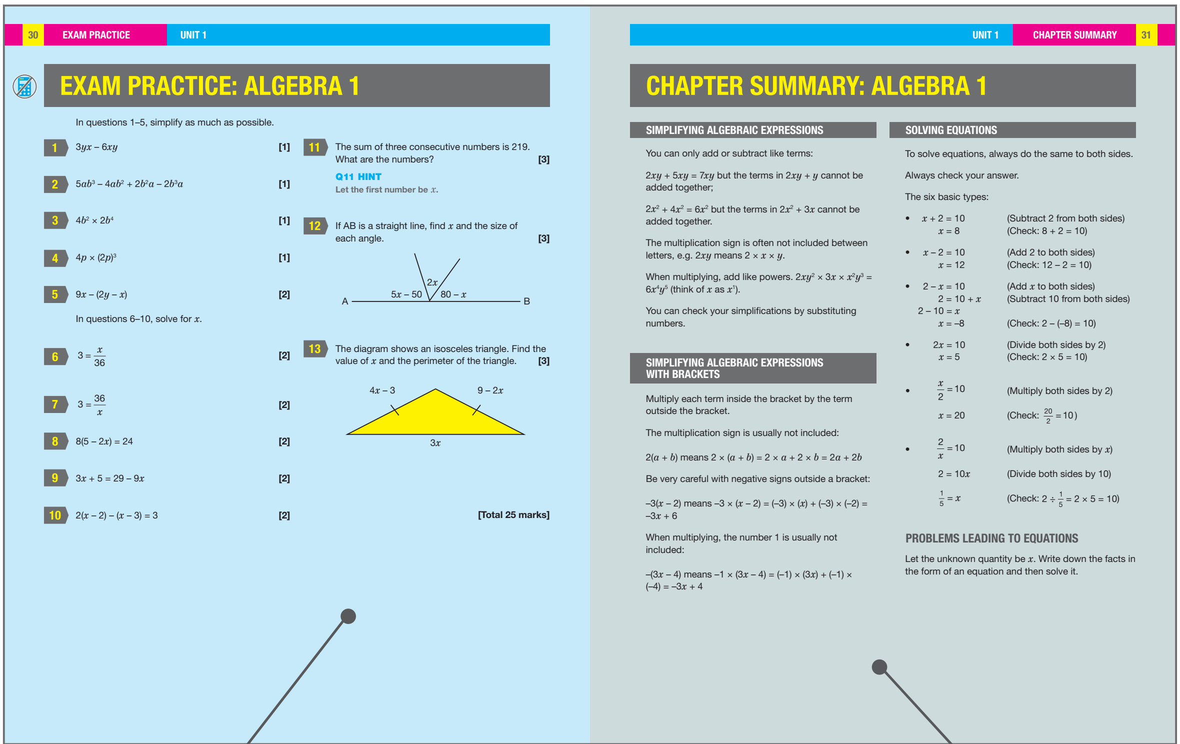
Starred exercises work towards grades 6–9.



Language is graded for speakers of English as an additional language (EAL), with advanced Maths-specific terminology highlighted and defined in the glossary at the back of the book.

Non-starred exercises work towards grades 1–6

More difficult questions appear at the end of some exercises and are identified by green question numbers.



Exam Practice tests cover the whole chapter and provide quick, effective feedback on your progress.

Chapter Summaries state the most important points of each chapter.

ASSESSMENT OVERVIEW

The following tables give an overview of the assessment for this course.

We recommend that you study this information closely to help ensure that you are fully prepared for this course and know exactly what to expect in the assessment.

PAPER 1	PERCENTAGE	MARK	TIME	AVAILABILITY
HIGHER TIER MATHS A Written examination paper Paper code 4MA1/3H Externally set and assessed by Edexcel	50%	100	2 hours	January and June examination series First assessment June 2018
PAPER 2	**PERCENTAGE**	**MARK**	**TIME**	**AVAILABILITY**
HIGHER TIER MATHS A Written examination paper Paper code 4MA1/4H Externally set and assessed by Edexcel	50%	100	2 hours	January and June examination series First assessment June 2018

ASSESSMENT OBJECTIVES AND WEIGHTINGS

ASSESSMENT OBJECTIVE	DESCRIPTION	% IN INTERNATIONAL GCSE
AO1	Demonstrate knowledge, understanding and skills in number and algebra: • numbers and the numbering system • calculations • solving numerical problems • equations, formulae and identities • sequences, functions and graphs	57–63%
AO2	Demonstrate knowledge, understanding and skills in shape, space and measures: • geometry and trigonometry • vectors and transformation geometry	22–28%
AO3	Demonstrate knowledge, understanding and skills in handling data: • statistics • probability	12–18%

ASSESSMENT SUMMARY

The Edexcel International GCSE in Mathematics (Specification A) **Higher Tier** requires students to demonstrate application and understanding of the following topics.

NUMBER

- Use numerical skills in a purely mathematical way and in real-life situations.

ALGEBRA

- Use letters as equivalent to numbers and as variables.
- Understand the distinction between expressions, equations and formulae.
- Use algebra to set up and solve problems.
- Demonstrate manipulative skills.
- Construct and use graphs.

GEOMETRY

- Use the properties of angles.
- Understand a range of transformations.
- Work within the metric system.
- Understand ideas of space and shape.
- Use ruler, compasses and protractor appropriately.

STATISTICS

- Understand basic ideas of statistical averages.
- Use a range of statistical techniques.
- Use basic ideas of probability.

Students should also be able to demonstrate **problem-solving skills** by translating problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes.

Students should be able to demonstrate **reasoning skills** by

- making deductions and drawing conclusions from mathematical information
- constructing chains of reasoning
- presenting arguments and proofs
- interpreting and communicating information accurately.

CALCULATORS

Students will be expected to have access to a suitable electronic calculator for both examination papers. The electronic calculator to be used by students attempting **Higher Tier** examination papers (3H and 4H) should have these functions as a minimum:

$+, -, \times, \div, x^2, \sqrt{x}$, memory, brackets, x^y, $x^{\frac{1}{y}}$, $\bar{x}$, Σx, Σfx, standard form, sine, cosine, tangent and their inverses.

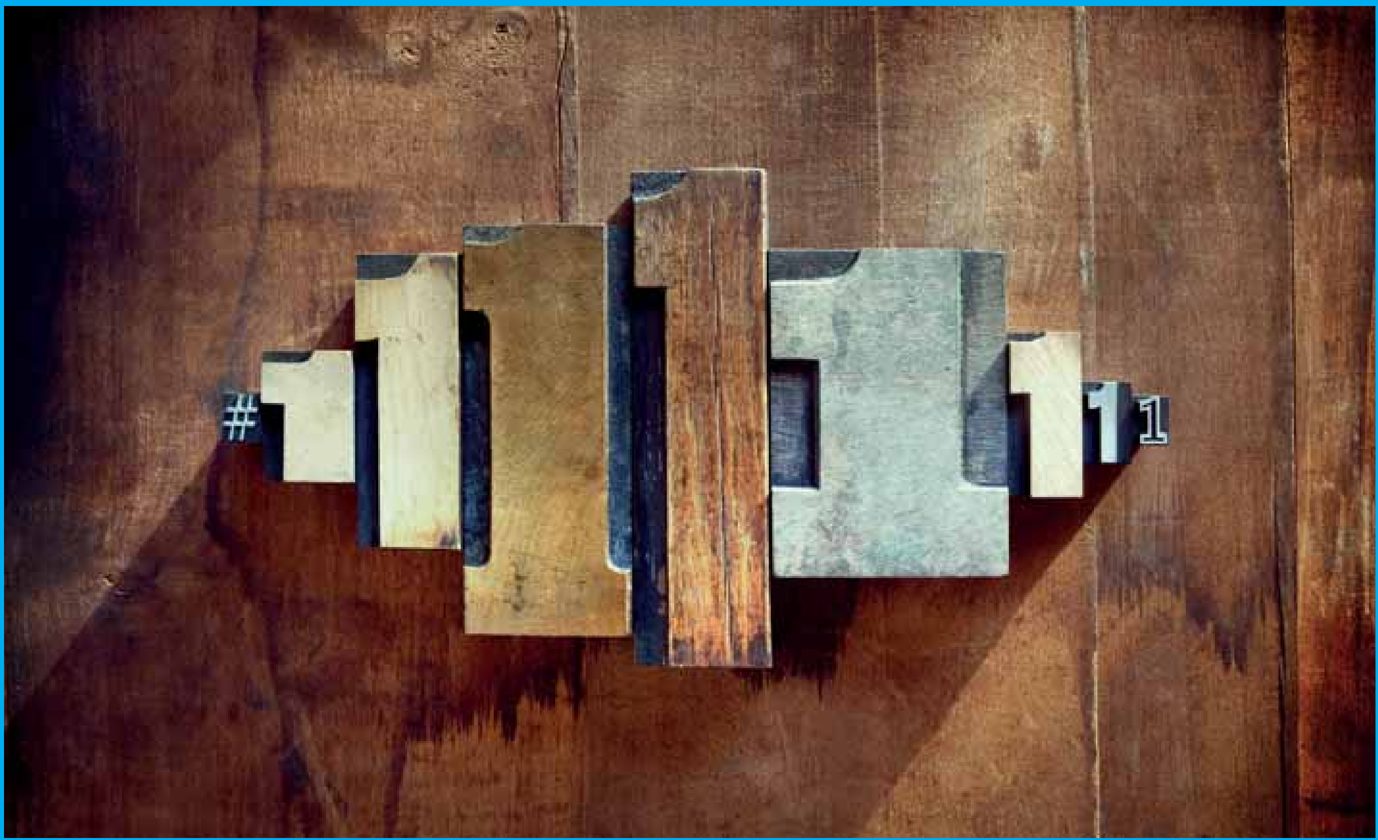
PROHIBITIONS

Calculators with any of the following facilities are prohibited in all examinations:

- databanks
- retrieval of text or formulae
- QWERTY keyboards
- built-in symbolic algebra manipulations
- symbolic differentiation or integration.

UNIT 1

1 is not a prime number. Any number multiplied by 1 is itself. Computer systems use the binary system that contains only two numbers (1 and 0) which represent numbers and instructions. It is also the most likely first number to appear in a list of numerical data as first described by Benford's Law.



NUMBER 1

The word fraction comes from the Latin 'fractio' which means 'to break'. Fractions in Ancient Egypt always had the top number as 1, such as $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, but it was very difficult to do calculations with them. In ancient Rome, fractions were written using words, not numbers, so calculations were also very difficult then. In India by about 500 AD fractions were being written with one number above the other but without a line. Around the year 1200 AD, the Ancient Arabs added the line to make fractions as we know them today.

LEARNING OBJECTIVES

- Add and subtract fractions and mixed numbers
- Multiply and divide fractions and mixed numbers
- Solve problems involving fractions

BASIC PRINCIPLES

- **Sign** of answer when multiplying or dividing:

$+ \times + = +$ $\quad + \times - = -$ $\quad - \times + = -$ $\quad - \times - = +$

$+ \div + = +$ $\quad + \div - = -$ $\quad - \div + = -$ $\quad - \div - = +$

- Finding **common factors**: Common factors of 12 and 8 are 2 and 4.
- Finding lowest **common denominator** when adding and subtracting fractions: Lowest common denominator of 6 and 4 is 12.
- The value of a fraction is not changed if the top and bottom are multiplied or divided by the same number:

$$\frac{1}{2} = \frac{3\times1}{3\times2} = \frac{3}{6} \qquad \frac{4}{10} = \frac{2\times2}{2\times5} = \frac{2}{5}$$

- Converting **mixed numbers** to fractions: $1\frac{2}{3} = \frac{5}{3}$

WORKING WITH FRACTIONS

Fraction calculations can be done on a calculator. In Unit 2, calculations are done with fractions like $\frac{x}{4}$. Since these cannot be done on a calculator, it is important that you can do fraction calculations without a calculator.

SIMPLIFYING FRACTIONS

A fraction has been simplified when the **numerator** (the top number) and the denominator (the bottom number) are expressed as whole numbers with no common factors.

EXAMPLE 1

SKILL: ANALYSIS

Simplify

a $\frac{28}{42}$ **b** $\frac{0.8}{1.6}$

a $\frac{28}{42} = \frac{2 \times 14}{2 \times 21} = \frac{2 \times 7}{3 \times 7} = \frac{2}{3}$

b $\frac{0.8}{1.6} = \frac{0.8 \times 10}{1.6 \times 10} = \frac{8}{16} = \frac{8 \times 1}{8 \times 2} = \frac{1}{2}$

Example 2 shows how to write decimals as fractions.

EXAMPLE 2

SKILL: ANALYSIS

Change **a** 0.4 **b** 0.025 to fractions.

a $0.4 = \frac{4}{10} = \frac{2}{5}$

b $0.025 = \frac{25}{1000} = \frac{5 \times 5}{5 \times 5 \times 40} = \frac{1}{40}$

To write a fraction as a decimal, divide the top number by the bottom number.

EXAMPLE 3

SKILL: ANALYSIS

Change **a** $\frac{2}{5}$ **b** $\frac{5}{8}$ to decimals

a $2 \div 5 = 0.4$ (using a calculator or **long division**)

b $5 \div 8 = 0.625$ (using a calculator or long division)

KEY POINTS

- Always simplify fractions.
- When working with mixed numbers, convert to **improper fractions** first.

EXERCISE 1

Simplify these.

1 ▶ $\frac{8}{12}$

2 ▶ $\frac{16}{24}$

3 ▶ $\frac{15}{45}$

4 ▶ $\frac{56}{84}$

5 ▶ $\frac{0.6}{1.2}$

6 ▶ $\frac{0.9}{2.7}$

Copy and complete this table, giving fractions in their lowest terms.

	FRACTION	DECIMAL
7 ▶	$\frac{4}{5}$	
8 ▶	$\frac{3}{8}$	
9 ▶		0.75
10 ▶		0.2

Change each of these to a mixed number.

11 ▶ $\frac{8}{3}$

12 ▶ $\frac{13}{4}$

13 ▶ $\frac{17}{5}$

14 ▶ $\frac{19}{7}$

Change each of these to an improper fraction.

15 ▶ $2\frac{1}{3}$ **16 ▶** $3\frac{3}{5}$ **17 ▶** $1\frac{5}{6}$ **18 ▶** $5\frac{6}{7}$

4

19 ▶ Write 18 minutes as a fraction of an hour in its simplest form.

20 ▶ Craig buys a ring for $500. He sells it for $750. Write the selling price as a fraction of the cost price in its simplest form.

EXERCISE 1*

3

Simplify and write each of these as a single fraction.

1 ▶ $\frac{6}{21}$ **3 ▶** $\frac{15}{90}$ **5 ▶** $\frac{0.7}{1.4}$

2 ▶ $\frac{14}{21}$ **4 ▶** $\frac{105}{165}$ **6 ▶** $\frac{1.2}{3.2}$

Copy and complete this table, giving fractions in their lowest terms.

	FRACTION	DECIMAL
7 ▶	$\frac{5}{16}$	
8 ▶	$\frac{3}{40}$	
9 ▶		0.35
10 ▶		0.375

Change each of these to a mixed number.

11 ▶ $\frac{13}{3}$ **12 ▶** $\frac{11}{5}$ **13 ▶** $\frac{23}{7}$ **14 ▶** $\frac{19}{4}$

Change each of these to an improper fraction.

15 ▶ $4\frac{2}{3}$ **16 ▶** $6\frac{3}{7}$ **17 ▶** $8\frac{2}{5}$ **18 ▶** $20\frac{8}{9}$

4

19 ▶ Elliot scores 65 out of 80 in a Maths test. Write this as a fraction in its simplest form.

20 ▶ Rendell cycles 42 km at an average speed of 18 km/hr. Find the time taken, giving your answer as a fraction of an hour in its simplest form.

MULTIPLYING FRACTIONS

If you do not know why one-half of one-third is the same as one-half multiplied by one-third, read the next example.

EXAMPLE 4

SKILL: PROBLEM SOLVING

Ella has a bar of chocolate. Her mother says she can eat one-half of one-third of the bar. How much does Ella eat?

When Ella unwraps the bar, she finds it has six squares.

One-third of the bar is two squares.

Half of this is one square.

So one-half of one-third of the bar is one square or one-sixth.

This is the same as one-half multiplied by one-third. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

ACTIVITY 1

SKILL: PROBLEM SOLVING

If Ella is eats one-half of two-thirds of the bar, how many squares does she eat?

Is this the same as $\frac{1}{2} \times \frac{2}{3}$?

Note that $\frac{1}{2} \times \frac{2}{3}$ can be calculated in two ways:

a Multiply top and bottom then **cancel down**: $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

b Cancel the 2s, then multiply: $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{1}{3}$

You can do the calculation in both ways, however the second method is usually more efficient.

Write mixed numbers as improper fractions before doing a calculation. If possible, divide by common factors before multiplying. Treat whole numbers as fractions, e.g. $5 = \frac{5}{1}$

EXAMPLE 5

Work out **a** $1\frac{2}{3} \times \frac{4}{5}$ **b** $5 \times \frac{3}{10}$

a $1\frac{2}{3} \times \frac{4}{5} = \frac{\cancel{5}}{3} \times \frac{4}{\cancel{5}} = \frac{4}{3} = 1\frac{1}{3}$

b $5 \times \frac{3}{10} = \frac{\cancel{5}}{1} \times \frac{3}{\cancel{10}_2} = \frac{3}{2} = 1\frac{1}{2}$

KEY POINTS

- The word ‘of’ means ‘multiplied by’.
- Convert mixed numbers into improper fractions before multiplying.
- If possible, divide by common factors before multiplying.
- Treat whole numbers as fractions, e.g. $5 = \frac{5}{1}$.

EXERCISE 2

Giving your answers as fractions in their lowest terms, work out

1 ▶ $\frac{5}{18} \times 3$

2 ▶ $\frac{4}{5} \times \frac{3}{8}$

3 ▶ $1\frac{3}{4} \times \frac{4}{7}$

4 ▶ $1\frac{1}{3} \times 1\frac{1}{2}$

5 ▶ $0.8 \times \frac{5}{16}$

6 ▶ $\frac{8}{9} \times 0.75$

7 ▶ $\frac{2}{5} \times \frac{3}{7} \times \frac{5}{6}$

8 ▶ $\frac{3}{7} \times \frac{5}{6} \times 1\frac{5}{9} \times 1\frac{3}{15}$

9 ▶ Three-sevenths of the songs in Riley’s music library are rock songs. Of the rock songs, seven-ninths feature a guitar solo. What fraction of the songs in Riley’s music library are rock songs featuring a guitar solo?

10 ▶ Imogen was doing her music practice for one-quarter of an hour. For two-thirds of that time she was practising her scales. For what fraction of an hour did she practise her scales?

EXERCISE 2*

Giving your answers as fractions in their lowest terms or as mixed numbers where appropriate, work out

1 ▶ $\frac{4}{5} \times \frac{15}{16}$

2 ▶ $1\frac{1}{4} \times \frac{1}{5}$

3 ▶ $3\frac{3}{8} \times 1\frac{1}{9}$

4 ▶ $8\frac{1}{4} \times 4\frac{4}{11}$

5 ▶ $\frac{3}{4} \times \frac{8}{7} \times \frac{21}{27} \times \frac{1}{4}$

6 ▶ $8\frac{2}{3} \times \frac{7}{13} \times 1\frac{2}{7}$

7 ▶ $\frac{a^2}{b} \times \frac{b}{a}$

8 ▶ $\frac{b}{a^2} \times \frac{b}{a} \times \frac{a^3}{b^2}$

9 ▶ Lucas divides his pizza into three equal pieces for himself and his two friends. His friend Teddy eats $\frac{5}{8}$ of his piece for lunch and a further $\frac{2}{5}$ of what remains for dinner. What fraction of the original pizza did Teddy eat for dinner?

10 ▶ In a factory, two-thirds of the floor area is taken up by the production line. Out of the remaining floor area, three-fifths is taken up by office space. The rest is warehouse space. The warehouse space occupies 2000 m^2. Work out the floor area of the production line.

DIVIDING FRACTIONS

To divide by a fraction, turn the fraction upside down and multiply. The next two examples explain this rule. The word 'reciprocal' is used for turning a fraction upside down.

EXAMPLE 6

SKILL: PROBLEM SOLVING

Half of Ella's chocolate bar is divided equally into three for three friends.
How much does each friend receive?

Half of Ella's bar is three squares of chocolate.

When divided in three, each friend receives one square or one-sixth of the original bar.

So $\frac{1}{2} \div 3 = \frac{1}{6}$

By writing 3 as $\frac{3}{1}$ you can see that the rule works: $\frac{1}{2} \div \frac{3}{1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$2 \div \frac{1}{3}$ means how many thirds are in two whole units.

EXAMPLE 7

SKILL: PROBLEM SOLVING

Ella has two bars of chocolate.

Both bars are divided into thirds.

How many blocks of chocolate are there?

One-third of a bar consists of two squares.

There are six blocks of one-third of a bar.

So $2 \div \frac{1}{3} = 6$

By writing 2 as $\frac{2}{1}$ you can see that the rule works: $\frac{2}{1} \div \frac{1}{3} = \frac{2}{1} \times \frac{3}{1} = \frac{6}{1} = 6$

Dividing by a fraction is the same as multiplying by the reciprocal of that fraction.

To find the reciprocal of a fraction, swap the numerator and the denominator.

EXAMPLE 8

Work out **a** $1\frac{2}{3} \div \frac{5}{6}$ **b** $9 \div 1\frac{1}{5}$ **c** $2\frac{2}{3} \div 4$

a $1\frac{2}{3} \div \frac{5}{6} = \frac{\cancel{5}}{\cancel{3}} \times \frac{{}^{2}\cancel{6}}{\cancel{5}} = \frac{2}{1} = 2$

b $9 \div 1\frac{1}{5} = \frac{9}{1} \div \frac{6}{5} = \frac{{}^{3}\cancel{9}}{1} \times \frac{5}{\cancel{6}_{2}} = \frac{15}{2} = 7\frac{1}{2}$

c $2\frac{2}{3} \div 4 = \frac{8}{3} \div \frac{4}{1} = \frac{{}^{2}\cancel{8}}{3} \times \frac{1}{\cancel{4}} = \frac{2}{3}$

KEY POINT

- To divide by a fraction, turn the fraction upside down and multiply.

EXERCISE 3

Giving your answers as fractions in their lowest terms or as mixed numbers where appropriate, work out

1 ▶ $\frac{3}{4} \div \frac{7}{8}$

2 ▶ $\frac{3}{10} \div \frac{4}{5}$

3 ▶ $\frac{12}{25} \div 4$

4 ▶ $9 \div \frac{3}{4}$

5 ▶ $6 \div 1\frac{1}{3}$

6 ▶ $1\frac{4}{5} \div 6$

7 ▶ $1\frac{1}{3} \div 2\frac{2}{5}$

8 ▶ $2\frac{1}{2} \div 2\frac{1}{4}$

9 ▶ Mia cuts up a piece of wood $4\frac{1}{2}$ m long into pieces measuring $\frac{3}{4}$ m long. How many pieces are there?

10 ▶ A bottle contains $2\frac{1}{4}$ litres of water. How many glasses of volume $\frac{3}{16}$ litre can it fill?

EXERCISE 3*

Giving your answers as fractions in their lowest terms or as mixed numbers where appropriate, work out

1 ▶ $\frac{2}{43} \div \frac{20}{21}$

2 ▶ $\frac{8}{15} \div \frac{6}{5}$

3 ▶ $16 \div \frac{2}{7}$

4 ▶ $3\frac{1}{9} \div 14$

5 ▶ $2\frac{1}{3} \div 2\frac{4}{5}$

6 ▶ $3\frac{3}{7} \div 2\frac{1}{7}$

7 ▶ $13\frac{1}{2} \div 2\frac{1}{4}$

8 ▶ $1\frac{3}{7} \div \frac{6}{35}$

9 ▶ A roll of ribbon is $32\frac{1}{2}$ cm long. How many pieces $1\frac{1}{4}$ cm long can be cut from the roll?

10 ▶ Dylan's cow produces $21\frac{1}{3}$ litres of milk per day. The milk is put into bottles with a volume of $2\frac{1}{3}$ litres. How many bottles does Dylan need each week to bottle all the milk?

ADDING AND SUBTRACTING FRACTIONS

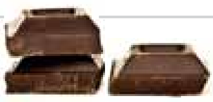
This can only be done if the denominators are the same.

EXAMPLE 9

SKILL: PROBLEM SOLVING

Ella eats one-half of her bar of chocolate and then eats a further third. What fraction of the bar has she eaten?

Half the bar is 3 squares.



One-third of the bar is 2 squares.

One-half plus one-third equals five-sixths or $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

EXAMPLE 10

Work out $\frac{3}{4}+\frac{1}{6}$

$\frac{3}{4}+\frac{1}{6}=\frac{9}{12}+\frac{2}{12}=\frac{9+2}{12}=\frac{11}{12}$

EXAMPLE 11

Work out $\frac{3}{4}-\frac{2}{5}$

$\frac{3}{4}-\frac{2}{5}=\frac{15}{20}-\frac{8}{20}=\frac{15-8}{20}=\frac{7}{20}$

EXAMPLE 12

Work out $3\frac{1}{3}-1\frac{3}{4}$

$3\frac{1}{3}-1\frac{3}{4}=\frac{10}{3}-\frac{7}{4}=\frac{40}{12}-\frac{21}{12}=\frac{40-21}{12}=\frac{19}{12}=1\frac{7}{12}$

KEY POINTS

- To add or subtract fractions, put them over a common denominator.
- Less work is needed if the common denominator is the lowest one.

EXERCISE 4

6 → 7

Giving your answers as fractions in their lowest terms or as mixed numbers where appropriate, work out

1 ▶ $\frac{2}{7}+\frac{4}{7}$

2 ▶ $\frac{4}{9}-\frac{1}{9}$

3 ▶ $\frac{5}{6}-\frac{1}{3}$

4 ▶ $\frac{11}{20}-\frac{3}{10}$

5 ▶ $\frac{3}{8}+\frac{7}{12}$

6 ▶ $\frac{5}{6}-\frac{3}{4}$

7 ▶ $3\frac{1}{4}+1\frac{1}{6}$

8 ▶ $4\frac{3}{5}-2\frac{1}{2}$

9 ▶ $2\frac{5}{6}+1\frac{3}{4}$

10 ▶ $3\frac{7}{8}+4\frac{1}{4}$

11 ▶ $5\frac{3}{10}-2\frac{11}{20}$

12 ▶ $36\frac{3}{8}-32\frac{7}{12}$

13 ▶ Li does one-quarter of her homework before dinner and a further one-third after dinner. What fraction of her homework remains undone?

14 ▶ A chemical consists of four compounds, A, B, C and D. $\frac{1}{6}$ is A, $\frac{2}{5}$ is B, $\frac{1}{10}$ is C and the rest is D. What fraction of the chemical is D?

EXERCISE 4*

6 → 7

Giving your answers as fractions in their lowest terms or as mixed numbers where appropriate, work out

1 ▶ $\frac{1}{3}+\frac{5}{12}$

2 ▶ $\frac{1}{4}+\frac{9}{20}$

3 ▶ $\frac{5}{6}-\frac{7}{30}$

4 ▶ $\frac{11}{15}-\frac{3}{20}$

5 ▶ $\frac{1}{5}+\frac{3}{10}+\frac{9}{20}$

6 ▶ $\frac{1}{4}+\frac{3}{20}-\frac{1}{40}$

7 ▶ $4\frac{1}{2}+3\frac{1}{6}$

8 ▶ $6\frac{2}{5}+7\frac{1}{3}$

9 ▶ $7\frac{2}{3}-1\frac{1}{6}$

10 ▶ $4\frac{7}{9}-3\frac{1}{3}$

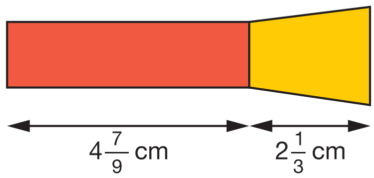
11 ▶ $7\frac{2}{3}-\frac{8}{9}$

12 ▶ $6\frac{1}{12}-4\frac{7}{10}$

13 ▶ Tonia and Trinny are twins. Their friends give them identical cakes for their birthday. Tonia eats $\frac{1}{8}$ of her cake and Trinny eats $\frac{1}{6}$ of her cake. How much cake is left?

14 ▶ A part has broken on a machine and needs to be replaced. The replacement part must be between $7\frac{1}{18}$ cm and $7\frac{1}{6}$ cm long in order to fit. The diagram shows the replacement part.

Will this part fit the machine? You must explain your answer.



ORDER OF OPERATIONS

The answer to 3 + 4 × 2 depends on whether the addition or multiplication is done first.

So that everybody gets the same answer to a calculation, there are rules for the order of operations. (Examples of operations: addition, subtraction, multiplication and division.)

The mnemonic BIDMAS will help you remember the correct order.

KEY POINT

• First	B	Brackets
• Second	I	Indices
• Third	DM	Division and/or Multiplication, working from left to right
• Fourth	AS	Addition and/or Subtraction, working from left to right

EXAMPLE 13

SKILL: INTERPRETATION

Evaluate $7 - 3 \div (5 - 2) \times 2^2 + 5$

The part of the expression being worked out at each step is highlighted in yellow.

$7 - 3 \div (5 - 2) \times 2^2 + 5 = 7 - 3 \div 3 \times 2^2 + 5$	Brackets
$7 - 3 \div 3 \times 2^2 + 5 = 7 - 3 \div 3 \times 4 + 5$	**Indices**
$7 - 3 \div 3 \times 4 + 5 = 7 - 1 \times 4 + 5$	Division and/or Multiplication, working l to r.
$7 - 1 \times 4 + 5 = 7 - 4 + 5$	Division and/or Multiplication, working l to r.
$7 - 4 + 5 = 3 + 5$	Addition and/or Subtraction, working l to r.
$3 + 5 = 8$	Addition and/or Subtraction, working l to r.

ACTIVITY 2

SKILL: INTERPRETATION

Without using your calculator, work out 2 + 3 × 4 and 3 × 4 + 2.

Check that your calculator gives the correct answer of 14 to 2 + 3 × 4 and to 3 × 4 + 2.

Use your calculator to check that 7 – 3 ÷ (5 – 2) × 22 + 5 = 8 (as in Example 13).

The line in a fraction acts like brackets. $\frac{1+2}{3}$ means $\frac{(1+2)}{3}$

EXAMPLE 14

SKILL: INTERPRETATION

Work out $\frac{16-4\times 3}{6\div 3\times 2}$

The part of the expression being worked out at each step is highlighted in yellow.

$$\frac{16-4\times 3}{6\div 3\times 2} \text{ means } \frac{(16-4\times 3)}{(6\div 3\times 2)}=\frac{(16-12)}{(2\times 2)}=\frac{4}{4}=1$$

EXERCISE 5

Work out the following.

1 ▶ 12 + 4 × 2

2 ▶ (12 + 4) × 2

3 ▶ 11 – 3^2

4 ▶ 12 – 2^2 × 3

5 ▶ $(8 - 3 \times 2)^2$

6 ▶ 5 + $(5 \times 2)^2$ ÷ 5

7 ▶ 3 + 2 ÷ (7 – 9) × (5 × 2 – 6)

8 ▶ $\frac{4+4^2}{6\div 3\times 2}$

9 ▶ Insert brackets in this expression to make it correct: 4 × 5 – 3 + 2 = 10

10 ▶ Insert brackets and symbols into this expression to make it correct: 7 5 3 = 6

EXERCISE 5*

Evaluate the following.

1 ▶ 4 + 6 × (2^2 + 5) ÷ 3 – 10

2 ▶ 2 – 5 ÷ (8 – 3) × 2 + 8

3 ▶ 125 ÷ $(7 \times 4 - 23)^2$ ÷ 5

4 ▶ $\frac{3}{4}\div\frac{9}{10}\times\frac{4}{5}\div\frac{2}{3}$

5 ▶ 1 + 10 ÷ 5 × 11 – 3^2 ÷ 3

6 ▶ (3 × 4 ÷ 2^2 + 3) × (6 ÷ 3 × 5 – 5 × 2 +1) – 5

7 ▶ $\frac{1+4\times 2}{6-1\times 2}\div\frac{12\div 2^2}{8\div 2\times 2}$

8 ▶ $\dfrac{\frac{2}{3}\times\frac{1}{4}+\frac{3}{4}\div\frac{9}{10}}{1+5\times\frac{3}{5}-\frac{6}{7}\div\frac{3}{7}}$

9 ▶ Insert brackets in this expression to make it correct:
8 – 2 + 1 × 5 – 3 = 2

10 ▶ Insert brackets and symbols in this expression to make it correct:
8 6 2 4 = 5

SIGNIFICANT FIGURES AND DECIMAL PLACES

If a piece of wood is to be cut 35.784 mm long then this measurement is too accurate to mark out and cut, so 35.784 would be **rounded** to a suitable **degree of accuracy**. Numbers can be rounded to a certain number of **significant figures** or **decimal places**.

SIGNIFICANT FIGURES (s.f.)

The first s.f. is the first non-zero digit in the number, counting from the left.

EXAMPLE 15

SKILL: INTERPRETATION

Highlight the first s.f. of the following numbers.

a 27 400 **b** 0.123 **c** 0.000 583

The first s.f. is highlighted in yellow.

a 27 400 **b** 0.123 **c** 0.000 583

For example, when rounding to 2 s.f., look at the third s.f. If this is greater than or equal to 5 then round the second figure up. If rounding to 3 s.f., look at the fourth s.f. and so on.

EXAMPLE 16

SKILL: INTERPRETATION

Write **a** 1361 **b** 1350 **c** 1349 **correct to** 2 s.f.

a 3rd s.f. is 6. $6 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 1361 = 1400 (2 s.f.)
(1361 is closer to 1400 than 1300)

b 3rd s.f. is 5. $5 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 1350 = 1400 (2 s.f.)
(1350 is midway between 1400 and 1300 but we round up in this case)

c 3rd s.f. is 4. $4 < 5$ so 3 is not rounded up $\Rightarrow$ 1349 = 1300 (2 s.f.)
(1349 is closer to 1300 than 1400)

EXAMPLE 17

SKILL: INTERPRETATION

Write **a** 0.001 361 **b** 0.001 35 **c** 0.001 349 correct to 2 s.f.

a 3rd s.f. is 6. $6 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 0.001 361 = 0.0014 (2 s.f.)
(0.001 361 is closer to 0.0014 than 0.0013)

b 3rd s.f. is 5. $5 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 0.001 35 = 0.0014 (2 s.f.)
(0.001 35 is midway between 0.0014 and 0.0013 but we round up in this case)

c 3rd s.f. is 4. $4 < 5$ so 3 is not rounded up $\Rightarrow$ 0.001 349 = 0.0013 (2 s.f.)
(0.001 349 is closer to 0.0013 than 0.0014)

DECIMAL PLACES (d.p.)

Count after the decimal point (going from left to right).
Rounding up or down follows the same rules as for s.f.

EXAMPLE 18

SKILL: INTERPRETATION

Write **a** 7.1361 **b** 0.135 **c** 0.0349 correct to 2 d.p.

a 3rd d.p. is 6. $6 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 7.1361 = 7.14 (2 d.p.)
(7.1361 is closer to 7.14 than 7.13)

b 3rd d.p. is 5. $5 \geq 5$ so 3 rounds up to 4 $\Rightarrow$ 0.135 = 0.14 (2 d.p.)
(0.135 is midway between 0.14 and 0.13 but we round up in this case)

c 3rd d.p. is 4. $4 < 5$ so 3 is not rounded up $\Rightarrow$ 0.0349 = 0.03 (2 d.p.)
(0.0349 is closer to 0.03 than 0.04)

This table shows π = 3.141 592 654… rounded to various degrees of accuracy.

DEGREE OF ACCURACY	SIGNIFICANT FIGURES	DECIMAL PLACES
5	3.1416	3.141 59
3	3.14	3.142
1	3	3.1

ACTIVITY 3

SKILL: INTERPRETATION

Use your calculator instruction book to find out how to:

- convert fractions to decimals and decimal to fractions
- round to a certain number of significant figures or decimal places.

Check by using the examples in this chapter.

KEY POINTS

- The first significant figure is the first non-zero digit in the number, counting from the left.
- For decimal places, count after the decimal point (going from left to right).
- If the next number is greater than or equal to 5, then round up.

EXERCISE 6

3 → 4

Write correct to 1 significant figure.

1 ▶ 783

2 ▶ 87 602

Write correct to 3 significant figures.

3 ▶ 3738

4 ▶ 80 290

Write correct to 2 significant figures.

5 ▶ 0.439

6 ▶ 0.555

Write correct to 3 significant figures.

7 ▶ 0.5057

8 ▶ 0.1045

Write correct to 2 decimal places.

9 ▶ 34.777

10 ▶ 0.654

Write correct to 1 decimal place.

11 ▶ 3.009

12 ▶ 9.09

13 ▶ The speed of light is 299 792 458 m/s.
Write this speed correct to **a** 3 s.f. **b** 6 s.f.

14 ▶ The **diameter** of a human hair is given as 0.0185 mm.
Write this diameter correct to **a** 2 d.p. **b** 2 s.f.

15 ▶ Pablo Picasso's 'Women of Algiers' sold at auction in New York for $179 365 000.
Write this price correct to 4 s.f.

16 ▶ The distance round the equator is 40 075 km. Write this distance correct to 1 s.f.

EXERCISE 6*

Write correct to 1 significant figure.

1 ▶ 10.49

2 ▶ 5049

Write correct to 3 significant figures.

3 ▶ 45.703

4 ▶ 89 508

Write correct to 2 significant figures.

5 ▶ 0.0688

6 ▶ 0.006 78

Write correct to 3 significant figures.

7 ▶ 0.049 549

8 ▶ 0.000 567 9

Write correct to 2 decimal places.

9 ▶ 8.997

10 ▶ 2.0765

Write correct to 1 decimal place.

11 ▶ 6.96

12 ▶ 78.1818

13 ▶ Write 0.000 497 5 correct to **a** 3 d.p. **b** 3 s.f.

14 ▶ Write $\sqrt{2}$ correct to **a** 6 d.p. **b** 6 s.f.

15 ▶ Only 10 bottles of a very exclusive and expensive perfume are made.
They are sold for the price of $12 721.89 per ounce.
Write this price correct to **a** 1 s.f **b** 1 d.p.

16 ▶ The Bohr radius is a physical constant of value 0.000 000 052 917 721 092 mm. Write the Bohr radius correct to **a** 7 d.p. **b** 7 s.f.

EXERCISE 7

REVISION

Give all answers, where appropriate, as fractions in their lowest terms.

1 ▶ Simplify **a** $\frac{12}{18}$ **b** $\frac{1.2}{18}$

2 ▶ Calculate **a** $2\frac{1}{6} \times \frac{3}{26}$ **b** $3\frac{1}{5} \div 1\frac{3}{5}$

3 ▶ Calculate **a** $2\frac{2}{5} + \frac{1}{4}$ **b** $2\frac{3}{4} - 1\frac{9}{10} + 1\frac{1}{5}$

4 ▶ Calculate **a** $10 - 3 \times 2$ **b** $6 - 3 \div 3 \times 4$ **c** $8 \div (3 - 1)^2 \times 2$

5 ▶ Insert brackets in this expression to make it correct: $12 \div 4 + 2 + 3 = 5$

6 ▶ Insert brackets and symbols in this expression to make it correct: 3 5 2 = 4

7 ▶ Write 12.000 497 5 correct to **a** 5 d.p. **b** 5 s.f

8 ▶ The age of the Earth is 4.543 billion years. Write 4.543 correct to **a** 1 d.p. **b** 1 s.f.

9 ▶ Geela has 20 litres of yoghurt that she wants to put into pots containing $1\frac{1}{4}$ litres each. How many pots can she fill?

10 ▶ Gill wears a device that counts the number of steps she takes every day. One day she did one-fifteenth of her steps before breakfast, a further half walking into town and another one-tenth walking round the supermarket.

a What fraction of her steps were not taken yet?

b That day the device recorded 12 000 steps. How many steps were not taken yet?

EXERCISE 7*

REVISION

Give all answers, where appropriate, as fractions in their lowest terms.

1 ▶ Simplify **a** $\frac{21}{63}$ **b** $\frac{0.21}{63}$

2 ▶ Calculate **a** $\frac{14}{15} \div 1\frac{2}{5}$ **b** $5\frac{1}{3} \div 4\frac{12}{13}$

3 ▶ Calculate **a** $2\frac{3}{4} - 1\frac{1}{5}$ **b** $1\frac{1}{8} - 1\frac{11}{12} + 1\frac{5}{6}$

4 ▶ Calculate **a** $25 \div (1 + 2^2)^2 \times 2$

b $5 + 12 \div 6 \times 2 - 18 \div 3^2$ **c** $\frac{2+2\times 2}{16-3\times 4} \div \frac{27 \div 3^2}{8 \div 2 \times 2}$

5 ▶ Insert brackets in this expression to make it correct: $2 \times 3 + 3 \div 3 = 3$

6 ▶ Insert brackets and symbols in this expression to make it correct: 7 2 2 3 = 6

7 ▶ Write 8.999 49 correct to **a** 3 d.p. **b** 3 s.f.

8 ▶ An important number in mathematics is Euler's number, e = 2.718 281 828 459 0…
Write Euler's number correct to **a** 8 s.f **b** 8 d.p.

9 ▶ Holly drinks $2\frac{4}{5}$ litres of water each day.
The water comes in $1\frac{2}{5}$ litre bottles.
How many bottles does Holly drink in a week?

10 ▶ Jake's computer has two hard drives that can store the same amount of data.
One drive is $\frac{3}{8}$ full while the other is $\frac{2}{5}$ full.

a What fraction of the total amount of storage space is empty?

b Each hard drive can store 750 gigabytes of data. Jake wants to download 150 gigabytes of data.
Does he have enough space?
Explain your answer.

EXAM PRACTICE: NUMBER 1

Give all answers where appropriate as fractions or mixed numbers in their lowest terms.

1 Simplify **a** $\frac{14}{42}$ **b** $\frac{140}{42}$ **c** $\frac{1.4}{42}$ **[3]**

2 Calculate **a** $\frac{5}{12} \times 1\frac{1}{15}$ **b** $5\frac{1}{4} \div \frac{7}{8}$ **[4]**

3 Calculate $\frac{4}{9} + 1\frac{3}{4} - 1\frac{1}{12}$ **[3]**

4 Calculate **a** $3 + 2 \times (1 + 4)^2$

b $\frac{1}{2} + \frac{1}{2} \div \frac{5}{6}$ **[4]**

5 A recent survey has found that the Great Wall of China is more than twice as long as was previously thought. Its length is now given as 21 196.18 km.

Write this length

a correct to 1 d.p. **b** correct to 1 s.f. **[2]**

6 The planning rules for a housing development state that $\frac{1}{3}$ of the houses should have three bedrooms, $\frac{3}{8}$ should have four bedrooms, $\frac{1}{24}$ should be executive homes and the rest should have two bedrooms.

a What fraction of the houses have two bedrooms?

b If 24 houses have two bedrooms, how many houses are on the development? **[5]**

7 Olivia's fish tank contains $42\frac{2}{3}$ litres of water. She is emptying it out using a scoop which holds $1\frac{1}{3}$ litres of water. How many full scoops will it take to empty the tank? **[4]**

[Total 25 marks]

CHAPTER SUMMARY: NUMBER 1

WORKING WITH FRACTIONS

Always simplify fractions to their lowest terms: $\frac{4}{6} = \frac{2\times\cancel{2}}{\cancel{2}\times 3} = \frac{2}{3}$.

The word 'of' means the same as 'multiplied by': $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$

Convert mixed numbers into improper fractions: $2\frac{1}{4} = \frac{9}{4}$

Treat whole numbers as fractions, e.g. $5 = \frac{5}{1}$

To divide by a fraction, turn the fraction upside down and multiply: $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

To add or subtract fractions, put them over a common denominator: $\frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$

ORDER OF OPERATIONS (BIDMAS)

• First	B	Brackets
• Second	I	Indices
• Third	DM	Division and/or Multiplication, working from left to right
• Fourth	AS	Addition and/or Subtraction, working from left to right

The part of the expression being worked out at each step is highlighted in yellow.

$5 + (2 + 1)^2 \times 4 = 5 + 3^2 \times 4$	Brackets
$5 + 3^2 \times 4 = 5 + 9 \times 4$	Indices
$5 + 9 \times 4 = 5 + 36$	Division and/or Multiplication
$5 + 36 = 41$	Addition and/or Subtraction

Note that calculators use the correct order of operations.

SIGNIFICANT FIGURES AND DECIMAL PLACES

The first significant figure is the first non-zero digit in the number, counting from the left.

The first s.f. is highlighted in yellow.

a 3400 **b** 0.367 **c** 0.00845

For decimal places, count after the decimal point (going from left to right).

The third d.p. is highlighted in yellow.

a 12.3456 **b** 0.00073

For example, when rounding to 2 s.f., look at the third s.f. If this is greater than or equal to 5 then round the second figure up. If rounding to 3 s.f., look at the fourth s.f. and so on.

2499 = 2000 (1 s.f.), 2499 = 2500 (2 s.f.), 0.2499 = 0.2 (1 d.p.), 0.2499 = 0.25 (2 d.p.)

ALGEBRA 1

Algebra may have begun in Egypt. The ancient Egyptians used the word 'aha', meaning 'heap', to stand for an unknown number. In the same way, we use a letter, such as x, today. The Ahmes Papyrus from Ancient Egypt around 1650BC contains problems that need a form of algebra to solve. They are believed to have been set as exercises for young mathematicians. These mathematical skills were probably essential for building the pyramids.

LEARNING OBJECTIVES

- Simplify algebraic expressions
- Expand brackets
- Solve equations in which the unknown appears on both sides

BASIC PRINCIPLES

- Algebra uses letters, often x, to stand for numbers.
- Algebraic expressions can be treated in the same way as number expressions.
- $x + 3$ means add three to the unknown number.
- $3x$ means 3 times the unknown number.
- x^2 means square the unknown number.

ACTIVITY 1

SKILL: PROBLEM SOLVING

Think of a number. Add 7 and then double the answer. Subtract 10, halve the result, and then subtract the original number.

Algebra can show you why the answer is always 2.

Think of a number:	x
Add 7:	$x + 7$
Double the result:	$2x + 14$
Subtract 10:	$2x + 4$
Halve the result:	$x + 2$
Subtract the original number:	2

Make two magic number tricks of your own, one like the example above and another that is longer. Check that they work using algebra. Then test them on a friend.

- Think of a number. Double it, add 12, halve the result, and then subtract the original number. Use algebra to find the answer.
  If you add a number other than 12, the answer will change. Work out the connection between the number you add and the answer.

SIMPLIFYING ALGEBRAIC EXPRESSIONS

ACTIVITY 2

SKILL: REASONING

Investigate the result when you substitute various values (positive or negative) for x in both of these expressions:

$x+1$ and $\dfrac{x^2+6x+5}{x+5}$

What is your conclusion? Which expression would you rather use?

EXAMPLE 1

Simplify $a + 3ab - 4ba$

$a + 3ab - 4ba = a - ab$

Note: $ab = ba$ so $3ab$ and $-4ba$ are **like terms** and can be simplified.

EXAMPLE 2

Simplify $3p^3 + 2p^2 - 2p^3 + 5p^2$

$3p^3 + 2p^2 - 2p^3 + 5p^2 = 3p^3 - 2p^3 + 5p^2 + 2p^2 = p^3 + 7p^2$

KEY POINTS

- You can only add or subtract like terms.
- $3ab+2ab=5ab$ but the terms in $3ab+b$ cannot be added together.
- $3a^2+2a^2=5a^2$ but the terms in $3a^2+2a$ cannot be added together.
- You can check your simplifications by substituting numbers.

EXERCISE 1

Simplify these as much as possible.

1 ▶ $9ab - 5ab$

2 ▶ $5xy + 2yx$

3 ▶ $4pq - 7qp$

4 ▶ $2xy + y - 3xy$

5 ▶ $x - 3x + 2 - 4x$

6 ▶ $7cd - 8dc + 3cd$

7 ▶ $6xy - 12xy + 2xy$

8 ▶ $4ab + 10bc - 2ab - 5cb$

9 ▶ $3ba - ab + 3ab - 5ab$

10 ▶ $4gh - 5jk - 2gh + 7$

11 ▶ $2p^2 - 5p^2 + 2p - 4p$

12 ▶ $2x^2y - xy^2 + 3yx^2 - 2y^2x$

EXERCISE 1*

Simplify these as much as possible.

1 ▶ $7xy + 5xy - 13xy$

2 ▶ $7ab - b - 3ab$

3 ▶ $2ab - 3ba + 7ab$

4 ▶ $12ab - 6ba + ba - 7ab$

5 ▶ $4ab + 10bc - ba - 7cb$

6 ▶ $q^2 + q^3 + 2q^2 - q^3$

7 ▶ $x^2 - 5x + 4 - x^2 + 6x - 3$

8 ▶ $5a^2 + a^3 - 3a^2 + a$

9 ▶ $h^3 + 5h - 3 - 4h^2 - 2h + 7 + 5h^2$

10 ▶ $3a^2b - 2ab + 4ba^2 - ba$

11 ▶ $0.7a^2b^3c - 0.4b^2a^3c + 0.3cb^3a^2 - 0.2a^3cb^2 + 0.3$

12 ▶ $2pq^2r^5 - pq^2r^4 - (r^4pq^2 - 2q^2r^5p)$

SIMPLIFYING ALGEBRAIC EXPRESSIONS WITH BRACKETS

EXAMPLE 3

Simplify $4r \times 5t$

$4r \times 5t = 20rt$

EXAMPLE 4

Simplify $(3b)^2 \times 3b$

$(3b)^2 \times 3b = 3b \times 3b \times 3b = 27b^3$

KEY POINTS

- The multiplication sign is often not included between letters, e.g. $3ab$ means $3 \times a \times b$.
- When multiplying, add like **powers** $3a^2b \times 2a^5b^4 \times a = 6a^8b^5$ (think of a as a^1).

EXERCISE 2

5 → 6

Simplify these.

1 ▶ $3 \times 2a$
2 ▶ $2x \times x$
3 ▶ $3x \times x^2$
4 ▶ $5a^3 \times 3a^2$
5 ▶ $2t \times 3s$
6 ▶ $4r \times s^2$
7 ▶ $2a^2 \times b^2$
8 ▶ $2y \times 2y \times y$
9 ▶ $2x^2 \times 3 \times 2x$
10 ▶ $(2a)^2 \times 5a$

EXERCISE 2*

7 → 9

Simplify these.

1 ▶ $8a \times a^2$
2 ▶ $5x^3 \times 3y^2 \times x$
3 ▶ $a^2 \times 2a^4 \times 3a$
4 ▶ $(3y)^2 \times 2y$
5 ▶ $6xy^2 \times 2x^3 \times 3xy$
6 ▶ $5abc \times 2ab^2c^3 \times 3ac$
7 ▶ $7x \times 2y^2 \times (2y)^2$
8 ▶ $2xy^2 \times 3x^2y + 4x^3y^3$
9 ▶ $x^2y^3 \times 3xy - 2x^3y^2$
10 ▶ $(2ab)^2 \times 5a^2b^4 - 2a^2b^5 \times 3a^2b$

EXPANDING BRACKETS

To simplify an expression with brackets, first multiply each term inside the bracket by the term outside the bracket, then simplify. This is called **expanding** the brackets.

EXAMPLE 5

Simplify $2(3 + x)$.

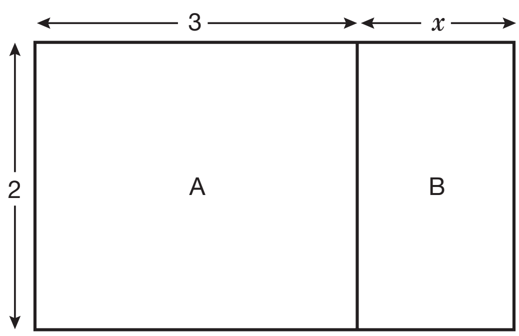
$2(3 + x) = 2 \times 3 + 2 \times x = 6 + 2x$

The diagram helps to show that $2(3 + x) = 6 + 2x$.

The area of the whole rectangle is $2(3 + x)$.

The area of rectangle A is 6.

The area of rectangle B is $2x$.



KEY POINTS

- Multiply each term inside the bracket by the term outside the bracket.
- The multiplication sign is usually left out:
  $3(x + y)$ means $3 \times (x + y) = 3 \times x + 3 \times y = 3x + 3y$
- Be very careful with negative signs outside a bracket:
  $-2(a - 3)$ means $-2 \times (a - 3) = (-2) \times (a) + (-2) \times (-3) = -2a + 6$
- When multiplying, the number 1 is usually left out:
  $-(2x + 3)$ means $-1 \times (2x + 3) = (-1) \times (2x) + (-1) \times (3) = -2x - 3$

EXERCISE 3

Remove the brackets and simplify these if possible.

1 ▶ $5(2 + 3a)$
2 ▶ $2(b - 4c)$
3 ▶ $-3(2a + 8)$
4 ▶ $-4(3 - x)$
5 ▶ $-(a - 2b)$
6 ▶ $3a + 2(a + 2b)$
7 ▶ $3(t - 4) - 6$
8 ▶ $7x - (x - y)$
9 ▶ $0.4(x - 3y) + 0.5(2x + 3y)$
10 ▶ $1.1(a + 3) - 5(3 - 0.2a)$

EXERCISE 3*

Remove the brackets and simplify these if possible.

1 ▶ $4(3m - 2)$
2 ▶ $2(x - y + z)$
3 ▶ $5(3a + b - 4c)$
4 ▶ $\frac{1}{2}(4x - 6y + 8)$
5 ▶ $5x - 3(2x - y)$
6 ▶ $0.4(2 - x) - (x + 3)$
7 ▶ $\frac{3}{4}(4x - 8y) - \frac{3}{5}(15x - 5y)$
8 ▶ $5x - 7y - 0.4(x - 2y + z)$
9 ▶ $0.3(2a - 6b + 1) - 0.4(3a + 6b - 1)$
10 ▶ $0.3x(0.2x - y) - 4y(x + 0.3y) + 0.5x(y - x)$

SOLVING EQUATIONS

If is often easier to solve mathematical problems using algebra. Let the unknown quantity be x and then write down the facts in the form of an equation. There are six basic types of equation:

$x + 3 = 12$ $\qquad x - 3 = 12$ $\qquad 3 - x = 12$

$3x = 12$ $\qquad \frac{x}{3} = 12$ $\qquad \frac{3}{x} = 12$

Solving an equation means having only x on one side of the equation.

EXAMPLE 6

Solve $x + 3 = 12$ for x.

$x + 3 = 12$ (Subtract 3 from both sides)
$x = 9$ (Check: $9 + 3 = 12$)

EXAMPLE 7

Solve $x - 3 = 12$ for x.

$x - 3 = 12$ (Add 3 to both sides)
$x = 15$ (Check: $15 - 3 = 12$)

EXAMPLE 8

Solve $3 - x = 12$ for x.

$3 - x = 12$ (Add x to both sides)

$3 = 12 + x$ (Subtract 12 from both sides)

$-12 + 3 = x$

$x = -9$ (Check: $3 - (-9) = 12$)

EXAMPLE 9

Solve $3x = 12$ for x.

$3x = 12$ (Divide both sides by 3)

$x = 4$ (Check: $3 \times 4 = 12$)

EXAMPLE 10

Solve $\frac{x}{3} = 12$ for x.

$\frac{x}{3} = 12$ (Multiply both sides by 3)

$x = 36$ (Check: $36 \div 3 = 12$)

EXAMPLE 11

Solve $\frac{3}{x} = 12$ for x.

$\frac{3}{x} = 12$ (Multiply both sides by x)

$3 = 12x$ (Divide both sides by 12)

$\frac{1}{4} = x$ (Check: $3 \div \frac{1}{4} = 12$)

KEY POINTS

- To solve equations, do the same thing to both sides.
- Always check your answer.

EXERCISE 4

3

Solve these for x.

1 ▶ $5x = 20$

2 ▶ $x + 5 = 20$

3 ▶ $x - 5 = 20$

4 ▶ $\frac{x}{5} = 20$

5 ▶ $3 = \frac{36}{x}$

6 ▶ $20 - x = 5$

7 ▶ $5x = 12$

8 ▶ $x - 3.8 = 9.7$

9 ▶ $3.8 = \frac{x}{7}$

10 ▶ $x + 9.7 = 11.1$

11 ▶ $13.085 - x = 12.1$

12 ▶ $\frac{34}{x} = 5$

EXERCISE 4*

3

Solve these for x.

1 ▶ $23.5 + x = 123.4$

2 ▶ $7.6x = 39$

3 ▶ $39.6 = x - 1.064$

4 ▶ $45.7 = \frac{x}{12.7}$

5 ▶ $7.89 = \frac{67}{x}$

6 ▶ $40.9 - x = 2.06$

EXAMPLE 12

Solve $3x - 5 = 7$ for x.

$3x - 5 = 7$ (Add 5 to both sides)

$3x = 12$ (Divide both sides by 3)

$x = 4$ (Check: $3 \times 4 - 5 = 7$)

EXAMPLE 13

Solve $4(x + 3) = 20$ for x.

$4(x + 3) = 20$ (Divide both sides by 4)

$x + 3 = 5$ (Subtract 3 from both sides)

$x = 2$ (Check: $4(2 + 3) = 20$)

EXAMPLE 14

Solve $2(x + 3) = 9$ for x.

$2(x + 3) = 9$ (**Multiply out** the bracket)

$2x + 6 = 9$ (Subtract 6 from both sides)

$2x = 3$ (Divide both sides by 2)

$x = \frac{3}{2}$ (Check: $2\left(\frac{3}{2}+3\right)=9$)

EXERCISE 5

5 → 6

Solve these for x.

1 ▶ $2x + 4 = 10$

2 ▶ $4x + 5 = 1$

3 ▶ $12x - 8 = -32$

4 ▶ $15x - 11 = -41$

5 ▶ $2(x + 3) = 10$

6 ▶ $5(x - 2) = 30$

7 ▶ $5 - x = 4$

8 ▶ $9 = 3 - x$

9 ▶ $12 = 2 - x$

10 ▶ $2(6 - 3x) = 6$

11 ▶ $3(6 - 2x) = 12$

12 ▶ $4(2 - x) = 16$

13 ▶ $6(3 - x) = 24$

14 ▶ $3(x - 5) = -13$

EXERCISE 5*

5 → 6

Solve these for x.

1 ▶ $5x - 3 = 17$

2 ▶ $27 = 3(x - 2)$

3 ▶ $7(x - 3) = -35$

4 ▶ $12(x + 5) = 0$

5 ▶ $9(x + 4) = 0$

6 ▶ $-7 = 9 + 4x$

7 ▶ $5 - 4x = -15$

8 ▶ $8 - 7x = -6$

9 ▶ $34 = 17(2 - x)$

10 ▶ $39 = 13(4 - x)$

11 ▶ $9(x + 4) = 41$

12 ▶ $5(10 - 3x) = 30$

13 ▶ $7(2 - 5x) = 49$

14 ▶ $6(4 - 7x) = 36$

EQUATIONS WITH x ON BOTH SIDES

EXAMPLE 15

Solve $7x - 3 = 3x + 5$ for x.

$7x - 3 = 3x + 5$	(Subtract $3x$ from both sides)
$7x - 3x - 3 = 5$	(Add 3 to both sides)
$4x = 5 + 3$	(Simplify)
$4x = 8$	(Divide both sides by 4)
$x = 2$	(Check: $7 \times 2 - 3 = 3 \times 2 + 5 = 11$)

EXAMPLE 16

Solve $5x + 6 = 3(10 - x)$ for x.

$5x + 6 = 3(10 - x)$	(Multiply out the bracket)
$5x + 6 = 30 - 3x$	(Add $3x$ and subtract 6 from both sides)
$5x + 3x = 30 - 6$	(Simplify)
$8x = 24$	(Divide both sides by 8)
$x = 3$	(Check: $5 \times 3 + 6 = 3(10 - 3) = 21$)

EXERCISE 6

Solve these for x.

1 ▶ $8x - 3 = 4x + 1$

2 ▶ $5x - 6 = 3x + 2$

3 ▶ $2x + 5 = 5x - 1$

4 ▶ $4x + 3 = 6x - 7$

5 ▶ $7x - 5 = 9x - 13$

6 ▶ $2x + 7 = 5x + 16$

7 ▶ $5x + 1 = 8 - 2x$

8 ▶ $14 - 3x = 10 - 7x$

9 ▶ $6 + 2x = 6 - 3x$

10 ▶ $8x + 9 = 6x + 8$

EXERCISE 6*

Solve these for x.

1 ▶ $3x + 8 = 7x - 8$

2 ▶ $7x + 5 = 5x + 1$

3 ▶ $5x + 7 = 9x + 1$

4 ▶ $4x + 3 = 7 - x$

5 ▶ $15x - 4 = 10 - 3x$

6 ▶ $5(x + 1) = 4(x + 2)$

7 ▶ $8(x + 5) = 10(x + 3)$

8 ▶ $3(x - 5) = 7(x + 4) - 7$

9 ▶ $3.1(4.8x - 1) - 3.9 = x + 1$

10 ▶ $8.9(x - 3.5) + 4.2(3x + 2.3) = 4.7x$

NEGATIVE SIGNS OUTSIDE BRACKETS

EXAMPLE 17

Solve $2(3x + 1) - (2x - 5) = 15$ for x.

$2(3x + 1) - (2x - 5) = 15$	(Remove brackets)
$6x + 2 - 2x + 5 = 15$	(Simplify)
$4x + 7 = 15$	(Subtract 7 from both sides)
$4x = 8$	(Divide both sides by 4)
$x = 2$	(Check: $2(3 \times 2 + 1) - (2 \times 2 - 5) = 15$)

KEY POINT

- $-(2x-5)$ means $-1\times(2x-5)=(-1)\times(2x)+(-1)\times(-5)=-2x+5$

EXERCISE 7

Solve these for x.

1 ▶ $3(x-2)-2(x+1)=5$

2 ▶ $4(x-1)-3(x+2)=26$

3 ▶ $3(2x+1)-2(2x-1)=11$

4 ▶ $9(x-2)-3(2x-3)=12$

5 ▶ $2(5x-7)-6(2x-3)=0$

6 ▶ $3(3x+2)-4(3x-3)=0$

7 ▶ $4(3x-1)-(x-2)=42$

8 ▶ $2(2x-1)-(x+5)=5$

9 ▶ $4(3-5x)-7(5-4x)+3=0$

10 ▶ $5(3x-2)-9(2+4x)-7=0$

EXERCISE 7*

Solve these for x.

1 ▶ $5(x-3)-4(x+1)=-11$

2 ▶ $9(x-2)-7(x+1)=-15$

3 ▶ $4(3x+5)-5(2x+6)=0$

4 ▶ $3(5x-4)-3(2x-1)=0$

5 ▶ $3(3x+1)-8(2x-3)+1=0$

6 ▶ $5(6x+2)-7(3x-5)-72=0$

7 ▶ $-2(x+3)-6(2x-4)+108=0$

8 ▶ $-3(x-2)-5(3x-2)+74=0$

9 ▶ $7(5x-3)-10=2(3x-5)-3(5-7x)$

10 ▶ $4(7+3x)-5(6-7x)+1=8(1+4x)$

PROBLEMS LEADING TO EQUATIONS

Let the unknown quantity be x. Write down the facts in the form of an equation and then solve it.

EXAMPLE 18

The sum of three **consecutive** numbers is 219. What are the numbers?

Let the first number be x. Then the next two numbers are $(x+1)$ and $(x+2)$.

$x+(x+1)+(x+2)=219$

$3x+3=219$

$3x=216$

$x=72$

So the three numbers are 72, 73 and 74. (Check: 72 + 73 + 74 = 219)

EXAMPLE 19

SKILL: REASONING

Find the value of x and the **perimeter** of this **isosceles triangle**.

As the triangle is isosceles

$4x+2=7x-4$

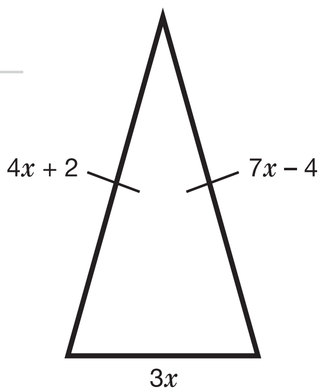
$2+4=7x-4x$

$6=3x$

$x=2$

Check: $4\times2+2=7\times2-4=10$

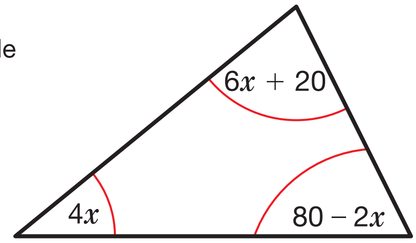
The sides are 10, 10 and 6 so the perimeter is 26.



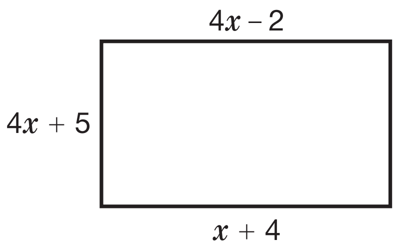
EXERCISE 8

1 ▶ The sum of two consecutive numbers is 477. What are the numbers? (Let the first number be x.)

2 ▶ Find x and the size of each angle in this triangle.



3 ▶ Find the value of x and the perimeter of this rectangle.



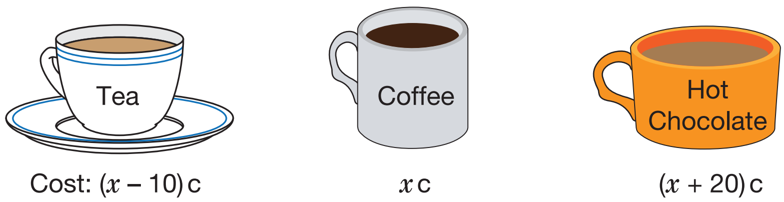
4 ▶ The result of doubling a certain number and adding 17 is the same as trebling (multiplying by 3) that number and adding 4. What is the number?

5 ▶ A kind teacher gives you 20 cents for every question you get right, but you have to pay the teacher 10 cents for every question you get wrong. After 30 questions you have made a profit of $1.80.

a Form an equation with x representing the number of questions you got right.

b Solve your equation to find how many questions you got right.

6 ▶ A cup of tea costs 10 cents less than a cup of coffee, while a cup of hot chocolate costs 20 cents more than a cup of coffee. Three cups of coffee, five cups of tea and two cups of hot chocolate cost $8.90.



a Form an equation with x representing the price of a cup of coffee.

b Solve your equation to find the price of a cup of coffee.

EXERCISE 8*

1 ▶ The sum of three consecutive even numbers is 222. Find the numbers.

2 ▶ John and Amelia have a baby daughter, Sonia. John is 23 kg heavier than Amelia, who is four times as heavy as Sonia. Their combined weight is 122 kg. How heavy is each person?

3 ▶ A father is three times as old as his son. In 14 years' time, he will be twice as old as his son. How old is the father now?

4 ▶ Lakshmi is trying to throw basketballs through hoops at a fair. If a ball goes through a hoop, she receives 50p, but if it does not she has to pay 20p for the shot. After 15 shots, Lakshmi finds she has made a profit of £1.20. How many times did Lakshmi successfully throw a ball through a hoop?

5 ▶ Aidan is doing a multiple-choice test with 20 questions. He scores 3 marks for a correct answer and loses 1 mark if the answer is incorrect. Aidan answers all the questions and scores 40 marks. How many questions has he answered correctly?

6 ▶ Freddie the frog is climbing up a well. Every day he climbs up 3 m but some nights he falls asleep and slips back 4 m. At the start of the sixteenth day, he has climbed a total of 29 m. On how many nights was he asleep?

EXERCISE 9

REVISION

Simplify these as much as possible.

1 ▶ $x + 2x + 3 - 5$

2 ▶ $3ba - ab + 3ab - 4ba$

3 ▶ $2a \times 3$

4 ▶ $2a \times a$

5 ▶ $a^2 \times a$

6 ▶ $2a^2 \times a^2$

7 ▶ $2a \times 2a \times a^2$

8 ▶ $7a - 4a(b + 3)$

9 ▶ $4(x + y) - 3(x - y)$

Solve these equations.

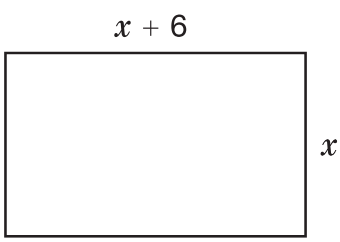
10 ▶ $2(x - 1) = 12$

11 ▶ $7x - 5 = 43 - 3x$

12 ▶ $5 - (x + 1) = 3x - 4$

13 ▶ Find three consecutive numbers whose sum is 438.

14 ▶ The perimeter of a rectangle is 54 cm. One side is x cm long and the other is 6 cm longer.



a Form an equation involving x.

b Solve the equation and write down the length of each of the sides.

EXERCISE 9*

REVISION

Simplify these as much as possible.

1 ▶ $6xy^2 - 3x^2y - 2y^2x$

2 ▶ $2xy^2 \times x^2y$

3 ▶ $p - (p - (p - (p - 1)))$

4 ▶ $xy(x^2 + xy + y^2) - x^2(y^2 - xy - x^2)$

Solve these equations.

5 ▶ $4 = \frac{x}{5}$

6 ▶ $4 = \frac{5}{x}$

7 ▶ $43 - 2x = 7 - 8x$

8 ▶ $1.3 - 0.3x = 0.2x + 0.3$

9 ▶ $0.6(x + 1) + 0.2(6 - x) = x - 0.6$

10 ▶ The length of a conference room is one and a half times its width. There is a carpet in the centre of the room. The length of the carpet is twice its width. This leaves a 3 m wide border around the edges of the carpet. Find the area of the carpet.

11 ▶ Two years ago, my age was four times the age of my son. Eight years ago, my age was ten times the age of my son. Find the age of my son now.

12 ▶ A river flows at 2 m/s. Juan's boat can travel twice as fast down the river as it can go up the river. How fast can the boat go in still water?

13 ▶ Matt wants to buy a television. If he pays cash, he gets a discount of 7%. If he pays with a loan he has to pay an extra 10% in interest. The difference between the two methods is $49.98. Find the cost of the television.

EXAM PRACTICE: ALGEBRA 1

In questions 1–5, simplify as much as possible.

1 $3yx - 6xy$ **[1]**

2 $5ab^3 - 4ab^2 + 2b^2a - 2b^3a$ **[1]**

3 $4b^2 \times 2b^4$ **[1]**

4 $4p \times (2p)^3$ **[1]**

5 $9x - (2y - x)$ **[2]**

In questions 6–10, solve for x.

6 $3 = \frac{x}{36}$ **[2]**

7 $3 = \frac{36}{x}$ **[2]**

8 $8(5 - 2x) = 24$ **[2]**

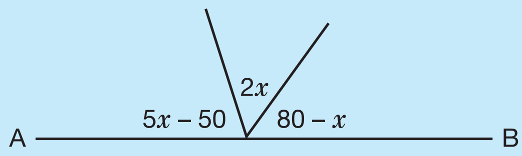
9 $3x + 5 = 29 - 9x$ **[2]**

10 $2(x - 2) - (x - 3) = 3$ **[2]**

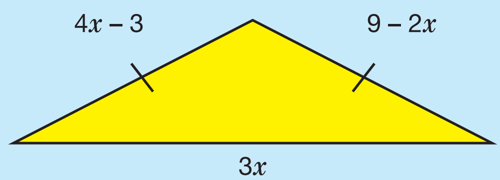
11 The sum of three consecutive numbers is 219. What are the numbers? **[3]**

Q11 HINT
Let the first number be x.

12 If AB is a straight line, find x and the size of each angle. **[3]**



13 The diagram shows an isosceles triangle. Find the value of x and the perimeter of the triangle. **[3]**



[Total 25 marks]

CHAPTER SUMMARY: ALGEBRA 1

SIMPLIFYING ALGEBRAIC EXPRESSIONS

You can only add or subtract like terms:

$2xy + 5xy = 7xy$ but the terms in $2xy + y$ cannot be added together;

$2x^2 + 4x^2 = 6x^2$ but the terms in $2x^2 + 3x$ cannot be added together.

The multiplication sign is often not included between letters, e.g. $2xy$ means $2 \times x \times y$.

When multiplying, add like powers. $2xy^2 \times 3x \times x^2y^3 = 6x^4y^5$ (think of x as x^1).

You can check your simplifications by substituting numbers.

SIMPLIFYING ALGEBRAIC EXPRESSIONS WITH BRACKETS

Multiply each term inside the bracket by the term outside the bracket.

The multiplication sign is usually not included:

$2(a + b)$ means $2 \times (a + b) = 2 \times a + 2 \times b = 2a + 2b$

Be very careful with negative signs outside a bracket:

$-3(x - 2)$ means $-3 \times (x - 2) = (-3) \times (x) + (-3) \times (-2) = -3x + 6$

When multiplying, the number 1 is usually not included:

$-(3x - 4)$ means $-1 \times (3x - 4) = (-1) \times (3x) + (-1) \times (-4) = -3x + 4$

SOLVING EQUATIONS

To solve equations, always do the same to both sides.

Always check your answer.

The six basic types:

- $x + 2 = 10$ (Subtract 2 from both sides)
  $x = 8$ (Check: $8 + 2 = 10$)
- $x - 2 = 10$ (Add 2 to both sides)
  $x = 12$ (Check: $12 - 2 = 10$)
- $2 - x = 10$ (Add x to both sides)
  $2 = 10 + x$ (Subtract 10 from both sides)
  $2 - 10 = x$
  $x = -8$ (Check: $2 - (-8) = 10$)
- $2x = 10$ (Divide both sides by 2)
  $x = 5$ (Check: $2 \times 5 = 10$)
- $\frac{x}{2} = 10$ (Multiply both sides by 2)
  $x = 20$ (Check: $\frac{20}{2} = 10$)
- $\frac{2}{x} = 10$ (Multiply both sides by x)
  $2 = 10x$ (Divide both sides by 10)
  $\frac{1}{5} = x$ (Check: $2 \div \frac{1}{5} = 2 \times 5 = 10$)

PROBLEMS LEADING TO EQUATIONS

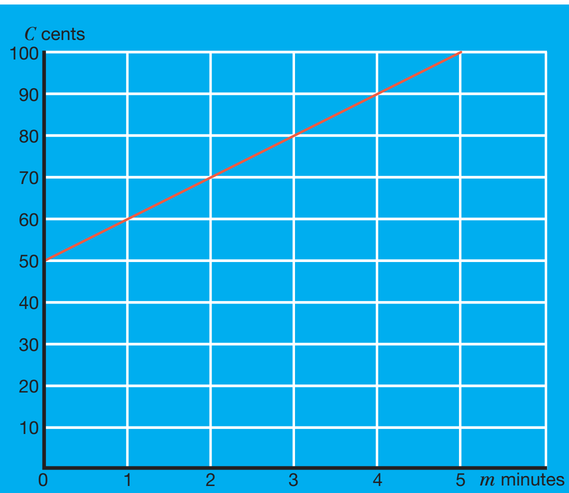
Let the unknown quantity be x. Write down the facts in the form of an equation and then solve it.

GRAPHS 1

The cost, C cents, of telephoning for m minutes is given by $C = 10m + 50$ and is shown on the graph of C against m.

The picture is much easier to understand than the algebraic expression.

Every time you graph an equation you are using the work of René Descartes (1596–1650), a French philosopher who connected algebra to geometry, therefore giving a picture to algebra. Graphs are sometimes called Cartesian graphs in his honour.

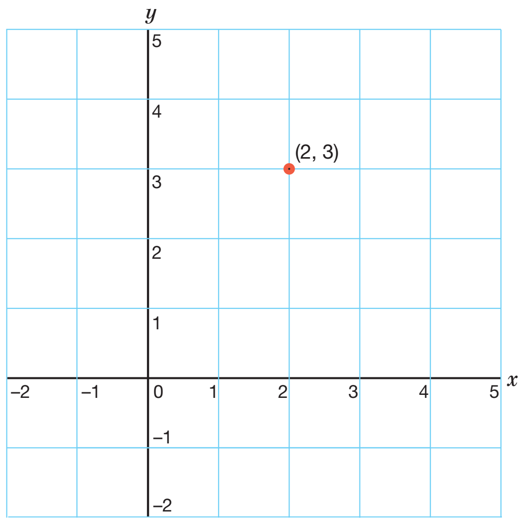


LEARNING OBJECTIVES

- Find the gradient of a line through two points
- Find the gradient and y-intercept of a straight line from its equation
- Compare two straight-line graphs using their equations
- Draw and interpret real-life graphs
- Plot graphs of straight lines with equations $ax + by = c$

BASIC PRINCIPLES

- Points on a graph are given by two numbers in brackets separated by a comma, for example (2, 3). All points are measured from the origin O.
- The x-axis is horizontal, the y-axis is vertical.
- The first number gives the distance from O in the x direction.
- The second number gives the distance from O in the y direction.
- These numbers can be positive or negative.



GRADIENT OF A STRAIGHT LINE



The pictures show some steep slopes. The slope of a line is its **gradient**.

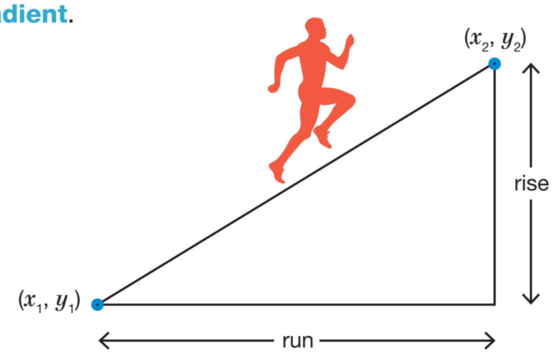
The larger the gradient, the steeper the slope.

The letter m is usually used for the gradient.

For a straight line $m = \frac{\text{change in the } y \text{ coordinates}}{\text{change in the } x \text{ coordinates}} = \frac{\text{'rise'}}{\text{'run'}}$

If the straight line joins the points (x_1, y_1) and (x_2, y_2) then 'rise' = $y_2 - y_1$ and 'run' = $x_2 - x_1$

The gradient is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



EXAMPLE 1

SKILL: PROBLEM SOLVING

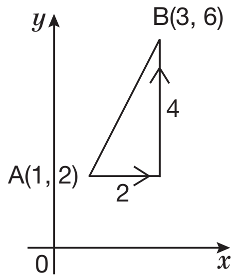
Find the gradient of the straight line joining A (1, 2) to B (3, 6).

First draw a diagram.

The gradient is $\frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ (a positive gradient).

Or use the formula with $x_1 = 1$, $y_1 = 2$, $x_2 = 3$, $y_2 = 6$

$m = \frac{6-2}{3-1} = \frac{4}{2} = 2$



EXAMPLE 2

SKILL: PROBLEM SOLVING

Find the gradient of the graph.

Choose two points on the graph and work out the rise and run.

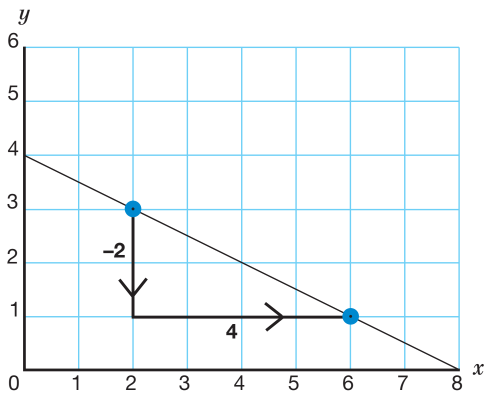
The gradient is $\frac{\text{rise}}{\text{run}} = \frac{-2}{4} = -\frac{1}{2}$ (a negative gradient).

Or use the formula. The two points chosen are (2, 3) and (6, 1) so

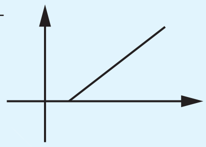
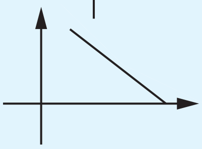
$m = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$

HINT

Do not use a ruler to measure the rise and run in case the x and y scales are different.



KEY POINTS

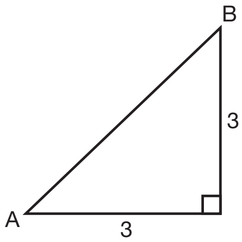
- Gradient m = $\frac{\text{'rise'}}{\text{'run'}}$
- Lines like this  have a positive gradient.
- Lines like this  have a negative gradient.
- Parallel lines have the same gradient.
- Always draw a diagram.

EXERCISE 1

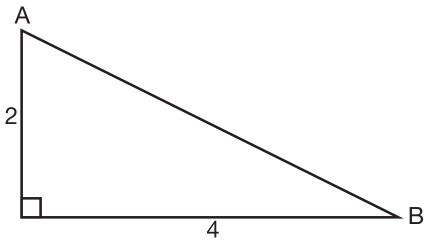
6

Find the gradient of the straight line joining A to B.

1 ▶



2 ▶



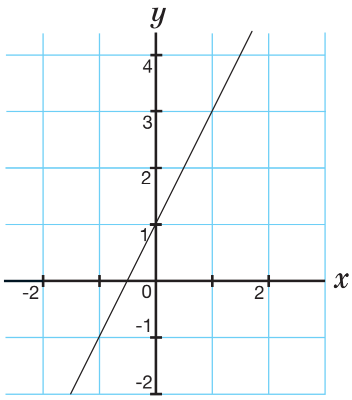
7

3 ▶ A is (1, 3) and B is (2, 6)

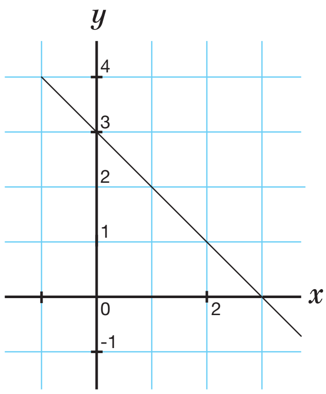
4 ▶ A is (–4, –1) and B is (4, 1)

5 ▶ A is (–2, 2) and B is (2, 1)

6 ▶ Find the gradient of the graph.



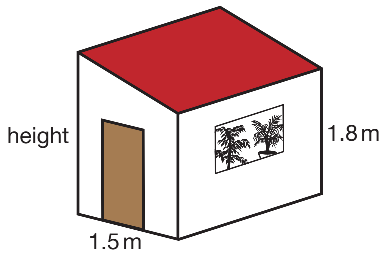
7 ▶ Find the gradient of the graph.



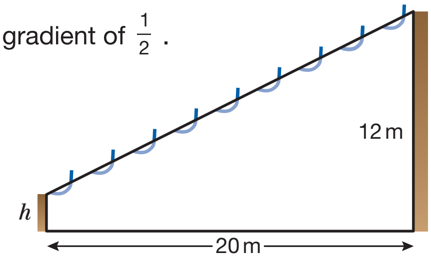
8 ▶ A ladder reaches 12 m up a vertical wall and has a gradient of 4. How far is the bottom of the ladder from the wall?

9 ▶ After take-off, an aeroplane climbs in a straight line with a gradient of $\frac{1}{5}$. When it has reached a height of 2000 m, how far has it gone horizontally?

10 ▶ The roof of this garden shed has a gradient of 0.35. Find the height of the shed.



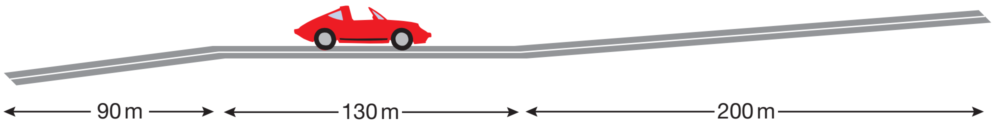
11 ▶ The seats at a football stadium are on a slope with gradient of $\frac{1}{2}$. What is the height (h) of the bottom seats?



12 ▶ A road has a gradient of $\frac{1}{15}$ for 90 m.

Then there is a horizontal section 130 m long.

The final section has a gradient of $\frac{1}{25}$ for 200 m.



a Find the total height gained from start to finish.
b What is the average gradient from start to finish?

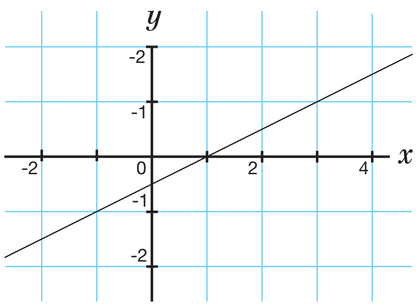
EXERCISE 1*

7

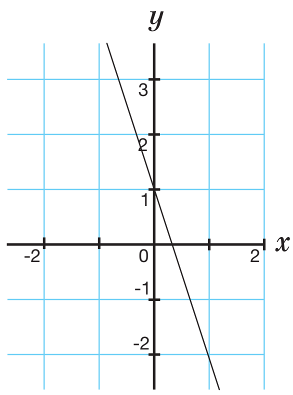
1 ▶ Find the gradient of the straight line joining A (–4, –1) to B (4, 2).

2 ▶ Find the gradient of the straight line joining A (–3, 2) to B (4, –4).

3 ▶ Find the gradient of the graph.



4 ▶ Find the gradient of the graph.

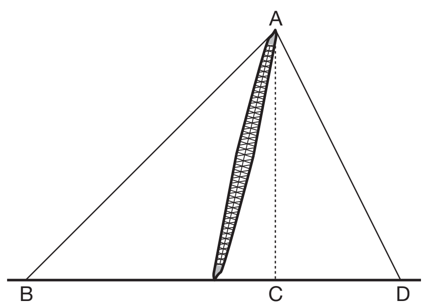


5 ▶ The line joining A (1, 4) to B (5, p) has a gradient of $\frac{1}{2}$. Find the value of p.

6 ▶ The masts for the Millennium Dome were held up during construction by wire ropes as shown in the diagram.

A is 106 m above the ground, C is vertically below A, the gradient of AB is 1 and CD is 53 m.

a Find the gradient of AD.
b Find the length of BD.

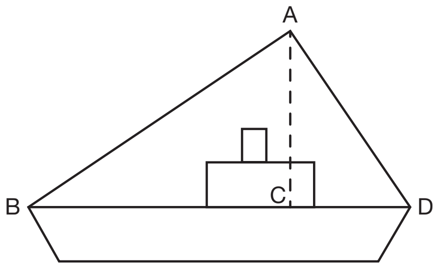


8

7 ▶ Antonio enjoys mountain biking. He has found that the maximum gradient which he can cycle up is 0.3 and the maximum gradient he can safely descend is 0.5. Antonio's map has a scale of 2 cm to 1 km with contours every 25 m. What is the minimum distance between the contours (lines on a map showing the height of land) on his map that allows him to go

a up-hill **b** down-hill?

8 ▶ A crane is lifting a boat suspended by wire ropes AB and AD. The point C is vertically below A, and BC measures 5 m.



a The gradient of AB is 0.8. How high is A above C?

b The gradient of AD is –1.25. What is the length of the boat?

8

9 ▶ Do the points (1, 2), (51, 27) and (91, 48) lie on a straight line? Give reasons for your answer.

10 ▶ Find an algebraic expression for the gradient of the straight line joining A (p, q) to B (r, s).

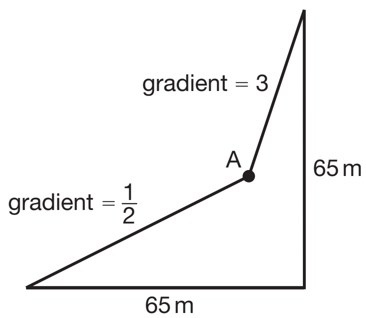
11 ▶ The line joining $(3, p)$ to $(7, -4p)$ is parallel to the line joining (–1, –3) to (3, 7). Find p.

12 ▶ The gradient of the line joining $(4, q)$ to (6, 5) is twice the gradient of the line joining (0, 0) to $(4, q)$. Find q.

10

13 ▶ One of the world's tallest roller coasters is in Blackpool, England. The maximum drop is 65 m over a horizontal distance of 65 m in two sections. The first section has a gradient of 3 and the second section has a gradient of $\frac{1}{2}$.

How high is the point A above the ground?



ACTIVITY 1

SKILL: REASONING

Find the gradient of the line AB.

Find the gradient of AB as the point B moves closer and closer to the point C.

Put your results in a table. What is the gradient of the horizontal line AC?

Find the gradient of AB as the point A moves closer and closer to the point C.

Put your results in a table. What is the gradient of the vertical line BC?