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Student Book

Edexcel IGCSE Mathematics B

D A Turner, I A Potts, W R J Waite, B V Hony

A PEARSON COMPANY



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Course structure

Preface



This book is written for students following the IGCSE Specification B for the Edexcel examination board, and contains numerous exercises and examples as well as much consolidation material. The authors are very experienced teachers, and most of the material has been thoroughly tested in the classroom.

The book contains ten **units** of work, with each unit containing the following five **sections**.

Number

Algebra

Functions and graphs

Mensuration, geometry and trigonometry

Sets, statistics, probability, vectors and matrices

In each **section** there are:

- **Concise explanations** and worked examples, with highlighted **Key Points**.
- **Activities** which lead pupils to discover mathematical principles for themselves.
- **Investigations** which prepare students for independent thought.
- **Parallel exercises** with the second exercise (starred) being more demanding than the first, allowing students to consolidate basic principles before attempting more difficult questions. Starred exercises are designed to challenge students working towards IGCSE grades A/A*. All exercises are carefully graded in difficulty to give students confidence. Real data is used wherever possible to make questions meaningful to students. At the end of some exercises there are more challenging questions that are marked in blue.
- **Revision exercises** at the end.

Each unit ends with a **multiple choice paper** and a **self-assessment paper**.

Unit 10 is a **consolidation unit**.

Simplifying fractions

Remember

A fraction has been simplified when the numerator (the top) and the denominator (the bottom) are expressed as whole numbers cancelled down as far as possible.

$$\bullet \frac{28}{42} = \frac{14}{21} = \frac{2}{3} \qquad \bullet \frac{0.8}{1.6} = \frac{8}{16} = \frac{1}{2}$$

Fractions are important when working with probabilities and ratios. They are also used in many other calculations in everyday life.

Remember

- To change a fraction or a decimal into a percentage, multiply it by 100.
- To change a percentage into a fraction, rewrite % as division by 100.
- To write a fraction as a decimal, divide the top number by the bottom number.

Exercise 1

Simplify these.

$$1 \frac{8}{12} \qquad 2 \frac{15}{45} \qquad 3 \frac{0.6}{1.2}$$

Copy and complete this table, giving the fractions in their lowest terms.

	Fraction	Decimal	Percentage
4		0.75	
5			25%
6	$\frac{3}{20}$		
7		0.35	

Change each of these to a mixed number.

$$8 \frac{8}{3} \qquad 9 \frac{17}{5}$$

Change each of these to an improper fraction.

$$10 2\frac{1}{3} \qquad 11 1\frac{5}{6}$$

Exercise 1*

Simplify and write each of these as a single fraction.

$$1 \frac{6}{21} \qquad 2 \frac{15}{90} \qquad 3 \frac{0.7}{1.4} \qquad 4 \frac{0.9}{12}$$

$$5 5 \times \frac{3}{18} \qquad 6 4 \times \frac{7}{42} \qquad 7 \frac{15}{27} \times 0.8 \qquad 8 0.3 \times \frac{7}{12}$$

Simplify and write each of these as an ordinary number.

$$9 68 \div 0.1 \qquad 10 765 \times 0.001 \qquad 11 \frac{7.8}{0.2}$$

$$12 \frac{36}{1.5} \qquad 13 25 \times \frac{105}{100} \qquad 14 46 \times \frac{91}{100}$$

Directed numbers

This section will remind you of how to work with negative numbers, and why it is important to do calculations in the correct order.

Remember

- Directed numbers

$$3 + (-4) = 3 - 4 = -1$$

$$3 - (-4) = 3 + 4 = 7$$

$$(-3) + (-4) = -3 - 4 = -7$$

$$(-3) - (-4) = -3 + 4 = 1$$

$$6 \times (-2) = -12$$

$$6 \div (-2) = -3$$

$$(-6) \div (-2) = 3$$

$$(-6) \times (-2) = 12$$

- Order of operations

The mnemonic **BIDMAS** may help you to remember the correct priority of operations when doing calculations working from left to right:

B Brackets

I Indices

Division and/or **M**ultiplication

Addition and/or **S**ubtraction

Exercise 2

Calculate these.

1 $(-5) + 10$

2 $10 - (-3)$

3 $17 + (-4)$

4 $(-7) - (-4)$

5 $(-4) \times 3$

6 $12 \div (-2)$

7 $\frac{(-16)}{8}$

8 $\frac{(-24)}{(-8)}$

9 $4 \times 3 - 2$

Exercise 2*

Calculate these.

1 $4 - 2 \times 3$

2 $20 \div 1 + 3$

3 $16 + 4 \div 2$

4 $16 + \frac{4}{2}$

5 $16 \div 2 + 4$

6 $\frac{16}{2} + 4$

7 $(2 \times 3)^2$

8 $(4 + 3)^2$

9 $(2 + 2) \times 6^2$

Percentages

Percentages are used to compare quantities. The unit of comparison is 100, and this is why the term 'per cent' is used ('per' means divide, and 'cent' means 100).

Example 1

Pacific Airlines increased ticket prices by 8%. Calculate the new price of a \$2450 ticket.

PACIFIC AIRLINES			
Boarding Closes 15 mins Before Boarding			
Carrier	PACIFIC AIRLINES		
Name	MISS K WIGLEY	Flight	PA 21
		Date	18 MAY
From	To	Seat	36A
AROUND THE WORLD-5 STOPS			
Price	\$2646,00	Boarding Time	21:05
		Date	12
		SEQ NBR	40
		Class	LUXURY FLYER
PACIFIC-AIRLINES.com			

$$100\% + 8\% = 108\%$$

$$\text{So multiplying factor} = \frac{108}{100} = 1.08$$

$$\begin{aligned} \text{New price} &= \$2450 \times 1.08 \\ &= \$2646 \end{aligned}$$

Example 2

In 1348–49, the population of England was 4 million.
The Black Death reduced the population by 37.5%. Find the new population.

$$100\% - 37.5\% = 62.5\%$$

$$\text{So multiplying factor} = \frac{62.5}{100} = 0.625$$

New population
= 4 million \times 0.625
= 2.5 million

Exercise 3

For Questions 1–3, find:

- 1 5% of 36
- 2 12% of 46 m
- 3 9% of 7.6
- 4 Increase \$30 by 6%.
- 5 Reduce \$50 by 9%.
- 6 Arun buys a car for \$15 000, and sells it for \$12 750. What is his percentage loss?
- 7 A bicycle is bought for \$250, and sold for \$275. What is the percentage profit?
- 8 Gembira throws the javelin 34 m. Then she improves this by 1.7 m. What is her percentage improvement?
- 9 A rare stamp is bought for \$7800 and increases in value by \$468.
Show that the increase in value is 6%.
- 10 Using percentages, comment on these figures.

	1980	2000	2010
Life expectancy of men in the Caribbean	70 years	72 years	75 years



Exercise 3*

For Questions 1–3, find:

- 1 1.5% of 50
- 2 5.7% of \$3000
- 3 7.5% of 700
- 4 Increase 67 km by 1.5%.
- 5 Decrease \$87 by 8%.
- 6 Marta buys a bicycle for \$350, reduced from \$402.50. What is her percentage saving?
- 7 A necklace is bought for \$34 and sold for \$38.25. Show that the percentage profit is 12.5%.
- 8 What is the percentage error if I use a value of $3\frac{1}{7}$ for π ?
- 9 A transatlantic airline ticket costs \$320 in the US, and \$360 in Europe. As a percentage, how much cheaper is the ticket in the US? As a percentage, how much more expensive is the ticket in Europe?
- 10 Use the data below to calculate the percentage difference due to gender and comment.

Events (world records)	1975	1985	1995	2005
Men's 100 m	9.95 s	9.93 s	9.84 s	9.77 s
Women's 100 m	11.07 s	10.76 s	10.49 s	10.49 s
Men's 1500 m	3 min 32.2 s	3 min 29.7 s	3 min .82 s	3 min 26.00 s
Women's 1500 m	4 min 1.4 s	3 min 52.4 s	3 min 50.46 s	3 min 50.46 s



Standard form (positive indices)

You can write the very large number 100 000 000 more simply as 1×10^8 using **standard form**. All numbers can be written in standard form, for example:

$$2904 = 2.904 \times 1000 = 2.904 \times 10^3$$

A standard form number can be converted back to an ordinary number:

$$5.6 \times 10^5 = 5.6 \times 100\,000 = 560\,000$$

Key Points

- In **standard form**, a million is written as 1×10^6 .
- **Standard form** is always written as $a \times 10^b$, where a is between 1 and 10, but never equal to 10, and b is an integer (a whole number).

Example 3

Convert 549 into standard form.

549 \rightarrow divide by 100 \rightarrow 5.49
So multiply by 100 to compensate.
 $549 = 5.49 \times 100 = 5.49 \times 10^2$

Example 4

Convert 7 670 000 into standard form.

7 6 7 0 0 0 0. move the decimal point
 $\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$ six places to make 7.67
So, $7\,670\,000 = 7.67 \times 10^6$

N.B. 'Moving the decimal point one place to the left' divides the number by 10.

Activity 1

In the human brain, there are about 100 000 000 000 neurons, and over the human lifespan 1000 000 000 000 000 neural connections are made.

- Write these numbers in standard form.
- Calculate the approximate number of neural connections made per second in an average human lifespan of 75 years.



Exercise 4

Calculate these, and write each answer in standard form.

- | | | | |
|----------------------|-----------------------|----------------------|-----------------------|
| 1 $10^2 \times 10^3$ | 2 $10^7 \times 10^3$ | 3 $10^1 \times 10^2$ | 4 $10^5 \times 10^8$ |
| 5 $10^5 \div 10^3$ | 6 $\frac{10^7}{10^4}$ | 7 $10^6 \div 10^3$ | 8 $10^{10} \div 10^9$ |

Write each of these in standard form.

- | | | | |
|-------|-----------|--------|-----------|
| 9 456 | 10 123.45 | 11 568 | 12 706.05 |
|-------|-----------|--------|-----------|

Write each of these as an ordinary number.

- | | | | |
|--------------------|-----------------------|----------------------|-----------------------|
| 13 4×10^3 | 14 4.09×10^6 | 15 5.6×10^2 | 16 7.97×10^6 |
|--------------------|-----------------------|----------------------|-----------------------|

17 The area of the surface of the largest known star is about 10^{15} square miles. The area of the surface of the Earth is about 10^{11} square miles. How many times greater is the star's area?

18 Calculate $(2 \times 10^4) \times (4.2 \times 10^5)$ and write the answer in standard form.

Exercise 4*

Write each of these in standard form.

- | | |
|----------|-----------------|
| 1 45 089 | 2 29.83 million |
|----------|-----------------|

Calculate these, and write each answer in standard form.

- | | | | |
|--------------------|-----------------------|-------------------------|--------------------|
| 3 10×10^2 | 4 $\frac{10^9}{10^4}$ | 5 $10^{12} \times 10^9$ | 6 $10^7 \div 10^7$ |
|--------------------|-----------------------|-------------------------|--------------------|

Calculate these, and write each answer in standard form.

7 $(5.6 \times 10^5) + (5.6 \times 10^6)$

8 $(3.6 \times 10^4) \div (9 \times 10^2)$

Calculate these, and write each answer in standard form.

9 $(4.5 \times 10^5)^3$

10 $10^{12} \div (4 \times 10^7)$

11 $10^9 - (3.47 \times 10^7)$

You will need the information in this table to answer Questions 12 and 13.

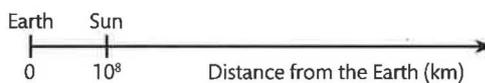
Celestial body	Approximate distance from Earth (km)
Sun	1.5×10^8
Saturn	1.5×10^9
Andromeda Galaxy (nearest major galaxy)	1.5×10^{19}



Copy and complete these sentences.

12 The Andromeda Galaxy is ... times further away from the Earth than Saturn.

13 To make a scale model showing the distances of the four bodies from the Earth, a student marks the Sun 1 cm from the Earth.



How far along the line should the other three celestial bodies be placed?

Significant figures and decimal places

It is often useful to simplify numbers by writing them either correct to so many **significant figures** (s.f.) or correct to so many **decimal places** (d.p.).

Example 5

Write 672 900 correct to 3 significant figures.

$672\,900 = 673\,000$ (to 3 s.f.)

4th s.f. = 9; $9 > 5$. So 2 rounds up to 3.

(672 900 is closer in value to 673 000 than to 672 000.)

Example 7

Write 6.4873 correct to 2 decimal places.

$6.4873 = 6.49$ (to 2 d.p.)

3rd d.p. = 7; $7 > 5$. So 8 rounds up to 9.

(6.4873 is closer in value to 6.49 than to 6.48.)

Example 6

Write 0.007645 correct to 2 significant figures.

$0.007645 = 0.0076$ (to 2 s.f.)

3rd s.f. = 4; $4 < 5$. So 6 is not rounded up to 7.

(0.007 645 is closer in value to 0.0076 than to 0.0077.)

Example 8

Write 23.428 correct to 1 decimal place.

$23.428 = 23.4$ (to 1 d.p.)

2nd d.p. = 2; $2 < 5$. So 4 is not rounded up to 5.

(23.428 is closer in value to 23.4 than to 23.5.)

Exercise 5

- Write 783 correct to 1 significant figure.
- Write 3783 correct to 3 significant figures.
- Correct 0.439 to 2 significant figures.
- Correct 0.5057 to 3 significant figures.

- 5 Write 34.777 to 2 decimal places.
- 6 Write 3.009 to 1 decimal place.
- 7 Write 105 678 in standard form correct to 1 s.f.
- 8 Write 98 765 in standard form correct to 1 s.f.

Exercise 5*

- 1 Write 10.49 correct to 1 significant figure.
- 2 Write 45.703 correct to 3 significant figures.
- 3 Correct 0.0688 to 2 significant figures.
- 4 Correct 0.049 549 to 3 significant figures.
- 5 Write 8.997 to 2 decimal places.
- 6 Write 6.96 to 1 decimal place.
- 7 Write 105 678 in standard form correct to 3 s.f.
- 8 Write 98 765 in standard form correct to 3 s.f.

Exercise 6 (Revision)

Write each of these as a fraction in its lowest terms.

- | | | |
|-------------------|----------------------------|----------------------------|
| 1 $\frac{4}{12}$ | 2 $\frac{4}{14}$ | 3 $\frac{6}{30}$ |
| 4 $\frac{12}{96}$ | 5 0.5 | 6 0.25 |
| 7 0.2 | 8 0.75 | 9 0.1 |
| 10 0.3 | 11 $2 \times \frac{3}{15}$ | 12 $\frac{4}{32} \times 6$ |

Calculate these.

- | | | |
|-------------------|------------------------|---------------------|
| 13 $5 \div 0.1$ | 14 $12 \div 0.2$ | 15 $3 \div 0.01$ |
| 16 $(-4) + 12$ | 17 $(-4) - 12$ | 18 $(-4) \times 12$ |
| 19 $(-4) \div 12$ | 20 $(-4) \times (-12)$ | |

Write the following percentages as fractions in their lowest terms.

- | | | |
|--------|--------|--------|
| 21 25% | 22 10% | 23 75% |
| 24 60% | 25 35% | |

26 Find 10% of 1500 m.

27 Find 15% of \$2400.

28 Increase 1500 m by 10%.

29 Decrease \$2400 by 15%.

30 A mobile phone from Pineapple Net is advertised at a 25% reduction in the New Year sales. Its price before New Year is \$120. What is the sale price?

31 Zack's pocket money is increased from €15 per week by 15%. How much does he receive per year after the increase?

32 Sami buys a computer game for \$36 after the cost has been reduced by 36%. What was the original cost?

Calculate these, and write each answer in standard form.

- | | | |
|---|---|---|
| 33 $(3 \times 10^4) \times (2 \times 10^6)$ | 34 $(8 \times 10^7) \div (2 \times 10^5)$ | 35 $(7 \times 10^7) \times (8 \times 10^8)$ |
|---|---|---|

Write each of these correct to 3 significant figures.

- 36** 1234 **37** 1235 **38** 1236
39 54321 **40** 54399

Write each of these correct to 3 decimal places.

- 41** 1.2344 **42** 1.2345 **43** 1.2305
44 1.2035 **45** 1.2007

Exercise 6* (Revision)

Write each of these as a fraction in its lowest terms.

- 1** $\frac{0.4}{4.4}$ **2** $3.6 \times \frac{3}{72}$ **3** $\frac{3}{70} \times 21$ **4** $\frac{1.2}{3.2} \times \frac{2.4}{7.2}$

- 5** Find 10% of 5% of 8400 g.
6 Alec buys a model boat for \$120, then sells it for \$75. What is his percentage loss?
7 Lonice buys a painting for \$1250, then sells it for \$1400. What is her percentage profit?
8 Leandra's fastest time for the 400 m is 70 seconds. In his next race he improves by 10% and in the race after that he improves from this time by 10% again.
a What is his new fastest time?
b What is his overall percentage improvement?
9 Nina was 1.25 m tall. One year later she was 10% taller and in the next year her height increased a further 12% from her new height.
a What is her height after both increases?
b What is her overall percentage height increase over the two years?
10 Leo has a salary of €120 000. Calculate his new salary if it is
a increased by 10% then decreased by 10%.
b increased by $x\%$ then decreased by $x\%$.

Calculate these.

- 11** $74.5 \div (0.1)^2$ **12** 74.5×0.001 **13** $74.5 \div 0.001$ **14** $74.5 \div (0.001)^2$

Calculate these and write each answer in standard form correct to 2 significant figures.

- 15** 4321×1234 **16** $(3.5 \times 10^8) \times (2.5 \times 10^6)$
17 $(3.6 \times 10^8) \div (7.2 \times 10^6)$ **18** $(3.6 \times 10^8) + (7.2 \times 10^6)$
19 $(3.6 \times 10^8) - (7.2 \times 10^6)$ **20** $(2.5 \times 10^5)^2$

Write each of these correct to 3 significant figures.

- 21** 0.2005 **22** 0.002005 **23** 3075.7 **24** 47555

Write each of these as a decimal correct to 3 decimal places.

- 25** 0.0785 **26** 0.0715 **27** $\left(\frac{1}{4}\right)^2$ **28** $\left(\frac{1}{3}\right)^2$

Unit 1: Algebra



Algebraic expressions contain letters which stand for numbers but can be treated in the same way as expressions containing numbers.

Activity 2

Think of a number. Add 7 and then double the answer. Subtract 10, halve the result, and then subtract the number you originally thought of. Algebra can show you why the answer is always 2.

Think of a number:	x
Add 7:	$x + 7$
Double the result:	$2x + 14$
Subtract 10:	$2x + 4$
Halve the result:	$x + 2$
Subtract the original number:	2

- Make up two magic number tricks of your own, one like the one above and another that is longer. Check that they work using algebra. Then test them on a friend.
- Think of a number. Double it, add 12, halve the result, and then subtract the original number.
 - Use algebra to find the answer.
 - If you add a number other than 12, the answer will change. Work out the connection between the number you add and the answer.

Simplifying algebraic expressions

You will often find it useful to simplify algebraic expressions before using them.

Exercise 7

Simplify these as much as possible.

- | | |
|---------------------------|-----------------------------|
| 1 $2a + 3a$ | 2 $6ab - 4ab$ |
| 3 $2a + 3b$ | 4 $3xy + 4yx$ |
| 5 $9ab - 5ab$ | 6 $5xy + 2yx$ |
| 7 $4pq - 7qp$ | 8 $2xy + y - 3xy$ |
| 9 $x - 3x + 2 - 4x$ | 10 $7cd - 8dc + 3cd$ |
| 11 $6xy - 12xy + 2xy$ | 12 $4ab + 10bc - 2ab - 5cb$ |
| 13 $3ba - ab + 3ab - 5ab$ | 14 $4gh - 5jk - 2gh + 7$ |

Exercise 7*

Simplify these as much as possible.

- | | |
|----------------------|---------------------------|
| 1 $2ab + 3ba$ | 2 $4xy - 2xz$ |
| 3 $4x + 3 - x$ | 4 $2y + 3z + y - z$ |
| 5 $7xy + 5xy - 13xy$ | 6 $7ab - b - 3ab$ |
| 7 $2ab - 3ba + 7ab$ | 8 $12ab - 6ba + ba - 7ab$ |

Key Points

You can only add or subtract **like** terms.

- $3ab + 2ab = 5ab$, but the terms in $3ab + b$ cannot be added together.
- $3a^2 + 2a^2 = 5a^2$, but the terms in $3a^2 + 2a$ cannot be added together.

You can check **your** simplifications by substituting numbers.

- 9 $4ab + 10bc - ba - 7cb$ 10 $q^2 + q^3 + 2q^2 - q^3$
 11 $x^2 - 5x + 4 - x^2 + 6x - 3$ 12 $5a^2 + a^3 - 3a^2 + a$
 13 $h^3 + 5h - 3 - 4h^2 - 2h + 7 + 5h^2$ 14 $3a^2b - 2ab + 4ba^2 - ba$

Simplifying algebraic products

The multiplication sign is often left out.

Key Point

$3ab$ means $3 \times a \times b$.

Exercise 8

Simplify these.

- 1 $3 \times 2a$ 2 $2x \times x$ 3 $3x \times x^2$ 4 $5a^3 \times 3a^2$
 5 $2t \times 3s$ 6 $4r \times s^2$ 7 $2a^2 \times b^2$ 8 $2y \times 2y \times y$
 9 $2x^2 \times 3 \times 2x$ 10 $(2a)^2 \times 5a$

Exercise 8*

Simplify these.

- 1 $2x \times 5y$ 2 $3x^2 \times 4x^3$ 3 $8a \times a^2$
 4 $2x \times 4x \times 3x$ 5 $5x^3 \times 3y^2 \times x$ 6 $a^2 \times 2a^4 \times 3a$
 7 $(3y)^2 \times 2y$ 8 $6xy^2 \times 2x^3 \times 3xy$ 9 $5abc \times 2ab^2c^3 \times 3ac$
 10 $7x \times 2y^2 \times (2y)^2$

Simplifying algebraic expressions with brackets

To simplify an expression with brackets, multiply each term inside the bracket by the term outside the bracket.

Example 1

Simplify $2(3 + x)$.

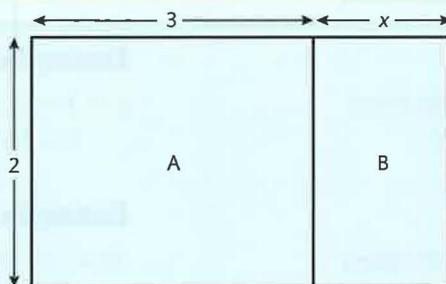
$$2(3 + x) = 2 \times 3 + 2 \times x = 6 + 2x$$

The diagram helps show that $2(3 + x) = 6 + 2x$.

The area of the whole rectangle is $2(3 + x)$.

The area of rectangle A is 6.

The area of rectangle B is $2x$.



Key Points

$$3(x + y) \text{ means } 3 \times (x + y) = 3 \times x + 3 \times y = 3x + 3y$$

Be very careful with negative signs outside a bracket.

$$-2(a - 3) \text{ means } -2 \times (a - 3) = (-2) \times (a) + (-2) \times (-3) = -2a + 6$$

When multiplying, the number 1 is usually left out.

$$-(2x + 3) \text{ means } -1 \times (2x + 3) = (-1) \times (2x) + (-1) \times (3) = -2x - 3$$

Exercise 9

Remove the brackets and simplify these if possible.

- | | | |
|-------------------|--------------------|------------------|
| 1 $4(x + y)$ | 2 $3(a - b)$ | 3 $5(2 + 3a)$ |
| 4 $2(b - 4c)$ | 5 $-3(2a + 8)$ | 6 $-4(3 - x)$ |
| 7 $-(a - 2b)$ | 8 $3a + 2(a + 2b)$ | 9 $3(t - 4) - 6$ |
| 10 $7x - (x - y)$ | | |

Exercise 9*

Remove the brackets and simplify these if possible.

- | | | |
|---|--------------------|------------------------------|
| 1 $5(x + 2y)$ | 2 $3(2x - 5y)$ | 3 $4(3m - 2)$ |
| 4 $2(x - y + z)$ | 5 $5(3a + b - 4c)$ | 6 $\frac{1}{2}(4x - 6y + 8)$ |
| 7 $5x - 3(2x - y)$ | 8 $-4(x + y - z)$ | 9 $0.4(2 - x) - (x + 3)$ |
| 10 $\frac{3}{4}(4x - 8y) - \frac{3}{5}(15x - 5y)$ | | |

Solving equations

It is often easier to solve mathematical problems using algebra. Let the unknown quantity be a letter, usually x , and then write down the facts in the form of an equation.

There are six basic types of equation:

$x + 3 = 12$	$x - 3 = 12$	$3 - x = 12$
$3x = 12$	$\frac{x}{3} = 12$	$\frac{3}{x} = 12$

Solving an equation means getting x on its own, on one side of the equation.

Remember

To solve equations, do the same thing to both sides.

Always check your answer.

Example 2

$$x + 3 = 12 \quad (\text{Subtract 3 from both sides})$$

$$x = 9 \quad (\text{Check: } 9 + 3 = 12)$$

Example 3

$$x - 3 = 12 \quad (\text{Add 3 to both sides})$$

$$x = 15 \quad (\text{Check: } 15 - 3 = 12)$$

Example 4

$$3 - x = 12 \quad (\text{Add } x \text{ to both sides})$$

$$3 = 12 + x \quad (\text{Subtract 12 from both sides})$$

$$-12 + 3 = x$$

$$x = -9 \quad (\text{Check: } 3 - (-9) = 12)$$

Example 5

$$3x = 12 \quad (\text{Divide both sides by 3})$$

$$x = 4 \quad (\text{Check: } 3 \times 4 = 12)$$

Example 6

$$\frac{x}{3} = 12 \quad (\text{Multiply both sides by 3})$$

$$x = 36 \quad (\text{Check: } 36 \div 3 = 12)$$

Example 7

$$\frac{3}{x} = 12 \quad (\text{Multiply both sides by } x)$$

$$3 = 12x \quad (\text{Divide both sides by 12})$$

$$\frac{1}{4} = x \quad (\text{Check: } 3 \div \frac{1}{4} = 12)$$

Exercise 10

Solve these for x .

- | | | |
|----------------------|------------------------|-----------------------|
| 1 $5x = 20$ | 2 $x + 5 = 20$ | 3 $x - 5 = 20$ |
| 4 $\frac{x}{5} = 20$ | 5 $3 = \frac{36}{x}$ | 6 $20 - x = 5$ |
| 7 $5x = 12$ | 8 $x - 3.8 = 9.7$ | 9 $3.8 = \frac{x}{7}$ |
| 10 $x + 9.7 = 11.1$ | 11 $13.085 - x = 12.1$ | 12 $\frac{34}{x} = 5$ |

Exercise 10*

Solve these for x , giving each answer correct to 3 significant figures.

- | | | |
|---------------------------|-------------------------|----------------------|
| 1 $23.5 + x = 123.4$ | 2 $7.6x = 39$ | 3 $39.6 = x - 1.064$ |
| 4 $45.7 = \frac{x}{12.7}$ | 5 $7.89 = \frac{67}{x}$ | 6 $40.9 - x = 2.06$ |

Example 8

$$\begin{aligned} 3x - 5 &= 7 && \text{(Add 5 to both sides)} \\ 3x &= 12 && \text{(Divide both sides by 3)} \\ x &= 4 && \text{(Check: } 3 \times 4 - 5 = 7) \end{aligned}$$

Example 9

$$\begin{aligned} 4(x + 3) &= 20 && \text{(Divide both sides by 4)} \\ x + 3 &= 5 && \text{(Subtract 3 from both sides)} \\ x &= 2 && \text{(Check: } 4(2 + 3) = 20) \end{aligned}$$

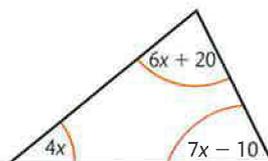
Example 10

$$\begin{aligned} 2(x + 3) &= 9 && \text{(Multiply out the bracket)} \\ 2x + 6 &= 9 && \text{(Subtract 6 from both sides)} \\ 2x &= 3 && \text{(Divide both sides by 2)} \\ x &= \frac{3}{2} && \text{(Check: } 2(\frac{3}{2} + 3) = 9) \end{aligned}$$

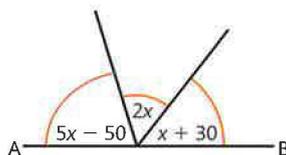
Exercise 11

Solve these for x .

- | | | | |
|-------------------|---------------------|--------------------|----------------------|
| 1 $2x + 4 = 10$ | 2 $4x + 5 = 1$ | 3 $12x - 8 = -32$ | 4 $2(x + 3) = 10$ |
| 5 $5(x - 2) = 30$ | 6 $5 - x = 4$ | 7 $9 = 3 - x$ | 8 $2(6 - 3x) = 6$ |
| 9 $4(2 - x) = 16$ | 10 $3(x - 5) = -13$ | 11 $9(x + 4) = 41$ | 12 $5(10 - 3x) = 30$ |
- 13 $7(2 - 5x) = 49$
- 14 The sum of two consecutive numbers is 477.
What are the numbers? (Let the first number be x .)
- 15 Find x and the size of each angle in this triangle.



- 16 AB is a straight line.
Find x and the size of each angle.



- 17 The formula for converting degrees Fahrenheit (F) to degrees Celsius (C) is $F = 32 + 1.8C$.
Find C when F is 5.

Exercise 11*

Solve these for x .

1 $5x - 3 = 17$

2 $27 = 3(x - 2)$

3 $7(x - 3) = -35$

4 $12(x + 5) = 0$

5 $-7 = 9 + 4x$

6 $5 - 4x = -15$

7 $34 = 17(2 - x)$

8 The sum of three consecutive even numbers is 222. Find the numbers.

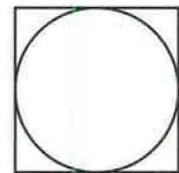
9 Zul and Amelia have a baby daughter, Shanise. Zul is 23 kg heavier than Amelia, who is four times heavier than Shanise. Their combined weight is 122 kg. How heavy is each person?

10 Emma buys some cans of cola at 28p each, and twice as many cans of orange at 22p each. She also buys ten fewer cans of lemonade than orange at 25p each. She spends £14.58. How many cans of cola did she buy?

11 Solve for x : $1.4(x - 3) + 0.2(2x - 1) = 0.1$.

12 Hama is training by running to a post across a field and then back. She runs the outward leg at 7 m/s and the return leg at 5 m/s. She takes 15.4 s. Find the distance to the post.

13 A piece of wire 30 cm long is cut into two pieces. One of these is bent into a circle, and the other is bent into a square enclosing the circle, as shown in the diagram. What is the diameter of the circle? (Remember that the circumference of a circle = $2\pi r$.)



14 Maureen sets off on a walk at 6 km/h. Ten minutes later, her brother John sets off after her on his bicycle at 15 km/h. How far must John go to catch up with Maureen?



Equations with x on both sides

Sometimes x appears on both sides of an equation.

Example 11

Solve this for x .

$$7x - 3 = 3x + 5 \quad (\text{Subtract } 3x \text{ from both sides})$$

$$7x - 3 - 3x = 5 \quad (\text{Add } 3 \text{ to both sides})$$

$$4x = 5 + 3$$

$$4x = 8 \quad (\text{Divide both sides by } 4)$$

$$x = 2 \quad (\text{Check: } 7 \times 2 - 3 = 3 \times 2 + 5)$$

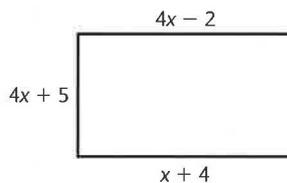
Exercise 12

Solve these for x .

1 $8x - 3 = 4x + 1$ 2 $2x + 5 = 5x - 1$ 3 $7x - 5 = 9x - 13$ 4 $2x + 7 = 5x + 16$

5 $5x + 1 = 8 - 2x$ 6 $14 - 3x = 10 - 7x$ 7 $6 + 2x = 6 - 3x$ 8 $8x + 9 = 6x + 8$

- 9 Find the value of x and the perimeter of this rectangle.



- 10 The result of adding 36 to a certain number is the same as multiplying that number by 5. What is the number?

Exercise 12*

Solve these for x .

- 1 $3x + 8 = 7x - 8$ 2 $7x + 5 = 5x + 1$ 3 $5x + 7 = 9x + 1$
 4 $4x + 3 = 7 - x$ 5 $15x - 4 = 10 - 3x$ 6 $5(x + 1) = 4(x + 2)$
 7 $8(x + 5) = 10(x + 3)$ 8 $3(x - 5) = 7(x + 4) - 7$ 9 $3.1(4.8x - 1) - 3.9 = x + 1$
- 10 If $\frac{x}{3} - 1$ is twice as large as $\frac{x}{4} - 3$, what is the value of x ?
- 11 A father is three times as old as his son. In 14 years' time, he will be twice as old as his son. How old is the father now?

Negative signs outside brackets

Key Point

$$-(2x - 5) \text{ means } -1 \times (2x - 5) = (-1) \times (2x) + (-1) \times (-5) = -2x + 5$$

Example 12

Solve this for x .

$$\begin{aligned} 2(3x + 1) - (2x - 5) &= 15 && \text{(Remove brackets)} \\ 6x + 2 - 2x + 5 &= 15 && \text{(Simplify)} \\ 6x - 2x &= 15 - 2 - 5 && \text{(Subtract 2 and 5 from both sides)} \\ 4x &= 8 && \text{(Divide both sides by 4)} \\ x &= 2 && \text{(Check: } 2(3 \times 2 + 1) - (2 \times 2 - 5) = 15) \end{aligned}$$

Exercise 13

Solve these for x .

- 1 $3(x - 2) - 2(x + 1) = 5$ 2 $3(2x + 1) - 2(2x - 1) = 11$
 3 $2(5x - 7) - 6(2x - 3) = 0$ 4 $4(3x - 1) - (x - 2) = 42$
 5 $4(3 - 5x) - 7(5 - 4x) + 3 = 0$

Exercise 13*

Solve these for x .

- 1 $5(x - 3) - 4(x + 1) = -11$ 2 $4(3x + 5) - 5(2x + 6) = 0$
 3 $3(3x + 1) - 8(2x - 3) + 1 = 0$ 4 $-2(x + 3) - 6(2x - 4) + 108 = 0$
 5 $7(5x - 3) - 10 = 2(3x - 5) - 3(5 - 7x)$



- 6** Anna is shooting at a target at a fair. If she hits the target she receives 50c, but if she misses she has to pay 20c for the shot. After 15 shots, Anna finds she has made a profit of \$1.20. How many hits has she had?
- 7** Michael is doing a multiple-choice test with 20 questions. He scores 3 marks for a correct answer and loses 1 mark if the answer is incorrect. Michael answers all the questions and scores 40 marks. How many questions has he got right?
- 8** Freddie the frog is climbing up a well. Every day he climbs up 3 m but some nights he falls asleep and slips back 4 m. At the start of the sixteenth day, he has climbed a total of 29 m. On how many nights was he asleep?

Exercise 14 (Revision)

Simplify these as much as possible.

- | | | |
|------------------------------------|---------------------------------|--------------------------------|
| 1 $x + 2x + 3 - 5$ | 2 $3ba - ab + 3ab - 4ba$ | 3 $2a \times 3$ |
| 4 $2a \times a$ | 5 $a^2 \times a$ | 6 $2a^2 \times a^2$ |
| 7 $2a \times 2a \times a^2$ | 8 $7a - 4a(b + 3)$ | 9 $4(x + y) - 3(x - y)$ |

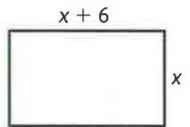
Solve these equations.

- 10** $2(x - 1) = 12$ **11** $7x - 5 = 43 - 3x$ **12** $5 - (x + 1) = 3x - 4$

- 13** Find three consecutive numbers whose sum is 438.

- 14** The perimeter of a rectangle is 54 cm. One side is x cm long, and the other is 6 cm longer.

- a** Form an equation involving x .
b Solve the equation, and write down the length of each of the sides.



Exercise 14* (Revision)

Simplify these as much as possible.

- | | |
|------------------------------------|---|
| 1 $6xy^2 - 3x^2y - 2y^2x$ | 2 $2xy^2 \times x^2y$ |
| 3 $p - (p - (p - (p - 1)))$ | 4 $xy(x^2 + xy + y^2) - x^2(y^2 - xy - x^2)$ |

Solve these equations.

- | | |
|--|------------------------------------|
| 5 $4 = \frac{x}{5}$ | 6 $4 = \frac{5}{x}$ |
| 7 $43 - 2x = 7 - 8x$ | 8 $1.3 - 0.3x = 0.2x + 0.3$ |
| 9 $0.6(x + 1) + 0.2(6 - x) = x - 0.6$ | |

- 10** The length of a conference room is one and a half times its width. A carpet that is twice as long as it is wide is placed in the centre of the room, leaving a 3 m wide border round the carpet.

Find the area of the carpet.

- 11** Two years ago, my age was four times that of my son. Eight years ago, my age was ten times that of my son. Find the age of my son now.
- 12** A river flows at 2 m/s. What is the speed through the water of a boat that can go twice as fast downstream as upstream?
- 13** Matt goes to buy a television. If he pays cash, he gets a discount of 7%. If he pays by instalments, he has to pay an extra 10% in interest. The difference between the two methods is \$2176. Find the cost of the television.

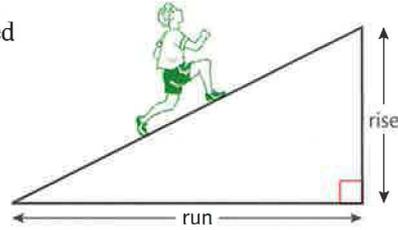


Gradient of a straight line

The slope of a line is its **gradient**. This is usually represented by the letter m .

For a straight line,

$$m = \frac{\text{change in the } y\text{-coordinates}}{\text{change in the } x\text{-coordinates}} = \frac{\text{'rise'}}{\text{'run'}}$$

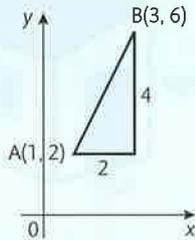


Example 1

Find the gradient of the straight line joining $A(1, 2)$ to $B(3, 6)$.

First draw a diagram.

Mark in the changes in coordinates.



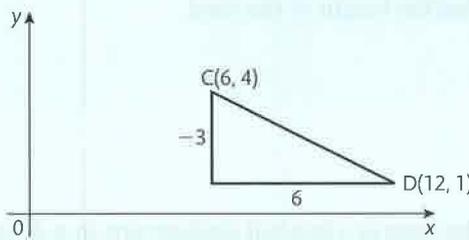
For AB, the change in the y -coordinate is $6 - 2 = 4$, and the change in the x -coordinate is $3 - 1 = 2$. The gradient is $\frac{4}{2} = 2$ (a positive gradient, running uphill).

Example 2

Find the gradient of the straight line joining $C(6, 4)$ to $D(12, 1)$.

First draw a diagram.

Mark in the changes in coordinates.



For CD, the change in the y -coordinate is $1 - 4 = -3$, and the change in the x -coordinate is $12 - 6 = 6$. The gradient is $\frac{-3}{6} = -\frac{1}{2}$ (a negative gradient, running downhill).

Key Points

- Gradient = $\frac{\text{'rise'}}{\text{'run'}}$
- Lines like this  have a *positive* gradient.
- Lines like this  have a *negative* gradient.
- Parallel lines have the same gradient.
- Always draw a diagram.

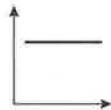
Investigate

- Find the gradient of the line AB.



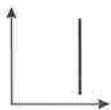
- Investigate the gradient of AB as point B moves closer and closer to point C. Tabulate your results. What is the gradient of the horizontal line AC?

What is the gradient of any horizontal line?



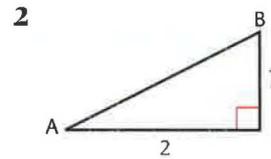
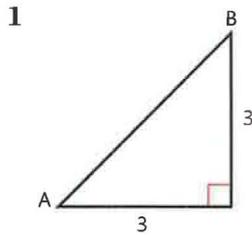
- Investigate the gradient of AB as point A moves closer and closer to point C. Tabulate your results. What is the gradient of the vertical line BC?

What is the gradient of any vertical line?



Exercise 15

For Questions 1–5, find the gradient of the straight line joining A to B when

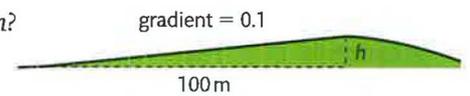


3 A is (1, 3) and B is (2, 6)

4 A is (−4, −1) and B is (4, 1)

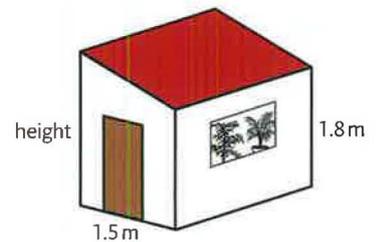
5 A is (−2, 2) and B is (2, 1)

6 A hill has a gradient of 0.1. What is the value of h ?

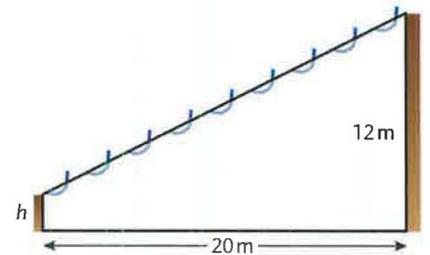


7 A ladder reaches 6 m up a vertical wall and has a gradient of 4. How far is the foot of the ladder from the wall?

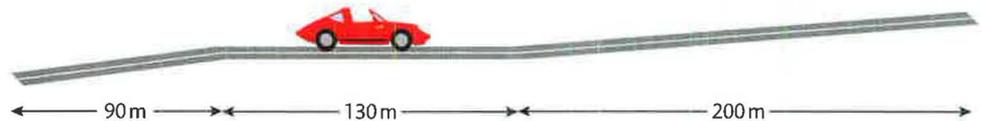
8 The roof of a lean-to garden shed has a gradient of 0.35. Find the height of the shed.



9 The seats at a football stadium are on a slope with a gradient of $\frac{1}{2}$. What is the height h of the bottom seats?



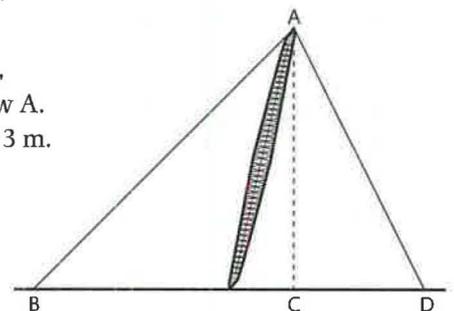
10 A road has a gradient of $\frac{1}{15}$ for 90 m. Then there is a horizontal section 130 m long. The final section has a gradient of $\frac{1}{25}$ for 200 m.



- Find the total height gained from start to finish.
- What is the average gradient from start to finish?

11 The masts for London's O₂ Arena were held up during erection by wire ropes. The top of a mast, A, is 106 m above the ground, and C is vertically below A. The gradient of one wire rope, AB, is 1, and CD is 53 m.

- Find the gradient of AD.
- Find the length of BD.



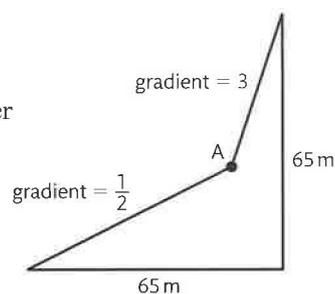
Exercise 15*

For Questions 1 and 2, find the gradients of the straight lines joining A to B when

- 1 A is $(-4, -1)$, B is $(4, 2)$ 2 A is $(-2, 4)$, B is $(2, 1)$
 3 The line joining A $(1, 4)$ to B $(5, p)$ has a gradient of 12. Find the value of p .

For Questions 4 and 5, find, by calculating gradients, whether or not the opposite sides of quadrilateral ABCD are parallel.

- 4 A is $(2, 1)$, B is $(14, 9)$, C is $(24, 23)$, D is $(10, 13)$
 5 A is $(2, 1)$, B is $(14, 7)$, C is $(20, 19)$, D is $(8, 13)$
 6 Alexander enjoys mountain biking. He has found that the maximum gradient which he can cycle up is 0.3, and the maximum gradient that he can safely descend is 0.5. His map has a scale of 2 cm to 1 km, with contours every 25 m.
 a What is the minimum distance between the contours on his map that allows him to go uphill?
 b What is the minimum distance between the contours on his map that allows him to go downhill?
 7 One of the world's tallest roller coasters is in Blackpool, England. It has a maximum height of 72 m, and gives white-knuckle rides at up to 140 km per hour. The maximum drop is 65 m over a horizontal distance of 65 m in two sections. The first section has a gradient of 3, and the second section has a gradient of $\frac{1}{2}$.
 How high above the ground is point A?
 8 The line joining $(3, p)$ to $(7, -4p)$ is parallel to the line joining $(-1, -3)$ to $(3, 7)$. Find p .



Straight-line graphs

Graphs of the form $y = mx + c$

Activity 3

- For each of these equations, copy and complete this table of values.

x	-2	0	2
y			

$$y = x + 1 \quad y = -x + 1 \quad y = 2x - 1$$

$$y = -2x + 1 \quad y = 3x - 1 \quad y = \frac{1}{2}x + 2$$

- Draw **one** set of axes, with the x -axis labelled from -2 to 2 and the y -axis from -7 to 5 . Plot the graphs of all six equations on this set of axes.
- Copy and complete this table.

Equation	Gradient	y -intercept
$y = x + 1$		
$y = -x + 1$		
$y = 2x - 1$		
$y = -2x + 1$		
$y = 3x + 1$		
$y = \frac{1}{2}x + 2$		
$y = mx + c$		

Can you see a connection between the number in front of x and the gradient?

The **y -intercept** is the value of y where the line crosses the y -axis.

Can you see a connection between the number at the end of the equation and the y intercept?

Key Points

The graph of $y = mx + c$ is a straight line with gradient m and y intercept c .

Sketching a straight line means showing the approximate position and slope of the line *without* plotting the line. If you know the gradient and intercept you can sketch the straight line easily.

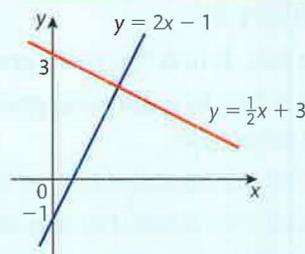
Example 3

Sketch these two lines.

$$y = 2x - 1 \quad y = -\frac{1}{2}x + 3$$

$y = 2x - 1$ is a straight line with gradient 2 and intercept -1 .

$y = -\frac{1}{2}x + 3$ is a straight line with gradient $-\frac{1}{2}$ and intercept 3.



Exercise 16

For Questions 1–8, write down the gradient and y intercept and then sketch the graph of the equation.

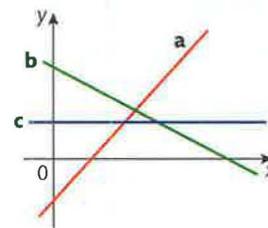
- 1** $y = x + 1$ **2** $y = \frac{1}{2}x + 4$ **3** $y = 3x + 5$ **4** $y = x - 7$
5 $y = \frac{1}{3}x + 2$ **6** $y = -\frac{1}{2}x + 5$ **7** $y = -\frac{1}{3}x - 2$ **8** $y = 4 - 2x$

For Questions 9 and 10, write down the equations of the lines with gradient

- 9** with gradient 2, passing through $(0, 1)$
10 with gradient -1 , passing through $(0, 2)$

For Questions 11–13, write down the equations of the lines that are parallel to

- 11** $y = 2x - 7$, passing through $(0, 4)$ **12** $y = 4 - 5x$, passing through $(0, -1)$
13 Write down possible equations for these sketch graphs.



Exercise 16*

For Questions 1–8, write down the gradient and y intercept and then sketch the graph of each equation.

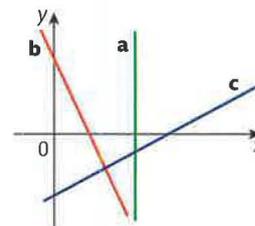
- 1** $y = 3x + 2$ **2** $y = -2x$ **3** $y = 5x + \frac{1}{2}$ **4** $y = -\frac{3}{4}$
5 $y = -3x + \frac{5}{2}$ **6** $y = 6x - \frac{3}{2}$ **7** $x = -1.5$ **8** $y = -\frac{2}{3}x - \frac{5}{3}$

For Questions 9 and 10, write down the equations of the lines with gradient

- 9** 2.5, passing through $(0, -2.3)$ **10** $\frac{1}{4}$, passing through $(4, 2)$

For Questions 11–13, write down the equations of the lines that are parallel to

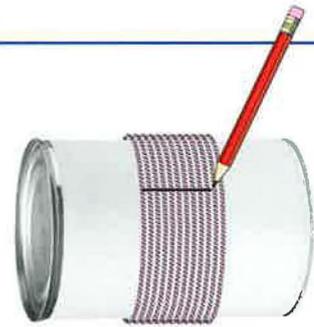
- 11** $2y = 5x + 7$, passing through $(0, -3.5)$ **12** $7x + 6y = 13$, passing through $(6, 7)$
13 Write down possible equations for these sketch graphs.



Activity 4

Equipment needed: a cylinder with a diameter of 5–10 cm (a drinks can or cardboard tube is ideal), a length of string about 30 times as long as the diameter of the cylinder, a ruler and graph paper.

- Wrap the string tightly around the cylinder, keeping the turns close together. Ask a friend to draw a straight line across the string while you hold the ends of the string.
- Unwind the string. Measure the distance of each mark from the first mark. Enter your results in a table.
- Plot these points on a graph of D against M . (Plot D on the vertical axis and M on the horizontal axis.) Draw the best straight line through these points.
- Calculate the gradient of the line, and then write down the equation of the line.
- The gradient should equal πd , where d is the diameter of the cylinder. Use your gradient to work out an estimate for π .
- Repeat the activity with various cylinders, and obtain further estimates for π .



Mark M	Distance D (cm)
1	0
2	
3	
4	
5	
6	

Graphs of the form $ax + by = c$

The graph of $3x + 4y = 12$ is a straight line.

The equation can be rearranged as $y = -\frac{3}{4}x + 3$, showing that the graph is a straight line with gradient $-\frac{3}{4}$ and y intercept $(0, 3)$.

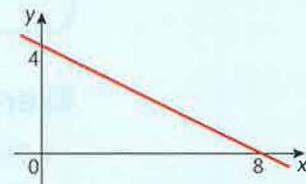
An easy way to draw or sketch this graph is to find where the graph crosses the axes.

Example 4

Sketch the graph of $x + 2y = 8$.

Substituting $y = 0$ gives $x = 8$, which shows that $(8, 0)$ lies on the line.

Substituting $x = 0$ gives $y = 4$, which shows that $(0, 4)$ lies on the line.



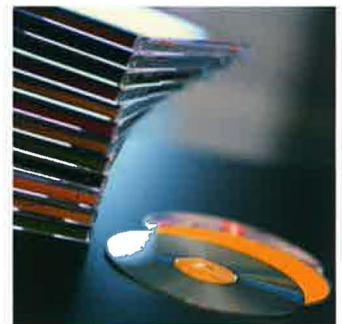
Exercise 17

For Questions 1–7, find where the graph crosses the axes and sketch the graph.

- 1 $x + y = 5$ 2 $3x + y = 6$ 3 $2x + y = 6$ 4 $3x + 2y = 12$
 5 $4x + 5y = 20$ 6 $x - 2y = 4$ 7 $4y - 3x = 24$

8 A firm selling CDs finds that the number sold (N thousand) is related to the price (£ P) by the formula $6P + N = 90$.

- Draw the graph of N against P for $0 \leq N \leq 90$ (the vertical axis should be the P axis, and the horizontal axis should be the N axis).
- Use your graph to find the price when 30 000 CDs are sold.
- Use your graph to find the number sold if the price of a CD is set at £8.
- Use your graph to find the price if 90 000 CDs are sold. Is this a sensible value?



Exercise 17*

For Questions 1–7, find where the graph crosses the axes and sketch the graph.

1 $4x + y = 12$

2 $x - 5y = 10$

3 $6x + 3y = 36$

4 $6x + 4y = 21$

5 $4x - 5y = 30$

6 $7y - 2x = 21$

7 $6x - 7y = -21$

- 8 Courtney has started playing golf. To try to reduce her handicap she has lessons with a professional. She keeps a record of her progress.

Week (W)	5	10	20	30
Handicap (H)	22	21	20	19

- Plot these points on a graph of H against W . Draw in the best straight line.
- What was Courtney's handicap before she started having lessons?
- Find the gradient and intercept of the line.
Write down the equation of the line in the form $ax + by = c$.
- To have a trial for the youth team, Courtney needs to have a handicap of less than 12. Use your equation to find how many weeks it will take Courtney to reduce her handicap to 12. Do you think this is a reasonable time?

Activity 5

Your aim is to find the equation of the straight line joining two points.

- Plot the points $A(1, 3)$ and $B(5, 5)$ on a graph. Find the gradient of AB .
- Calculate where the straight line passing through AB will intercept the y -axis.
- Write down the equation of the straight line passing through A and B .
- Use this method to find the equation of the straight line joining these pairs of points:
 $(-2, 1)$ and $(-1, 4)$ $(-3, 4)$ and $(6, 1)$ $(-2, -1)$ and $(4, 3)$

Exercise 18 (Revision)

- Find the gradient of the straight line joining A to B when
 - A is $(3, 4)$, B is $(5, 8)$
 - A is $(-1, 2)$, B is $(1, 0)$
- The foot of a ladder is 1.5 m from the base of a vertical wall. The gradient of the ladder is 3. How far does the ladder reach up the wall?
- Write down the gradient and y intercept of the graph of
 - $y = 3x - 2$
 - $y = -2x + 5$
- Write down the equations of the lines with
 - gradient 2, passing through $(0, -1)$
 - gradient -3 , passing through $(0, 2)$
- Sketch the following graphs.
 - $y = 2x - 3$
 - $y = 4 - x$
 - $2x + 5y = 10$
- Which of these lines are parallel?

$y = 2x + 4$	$x - 3y = 1$	$4x = 2y + 7$	$9y = 3x + 4$
$4x - 3y = 12$	$3x - 4y = 12$	$3y = 4x - 1$	$4y = 3x + 7$
- Find the gradients of the lines parallel to AB when
 - A is $(1, 2)$, B is $(4, 4)$
 - A is $(-2, 1)$, B is $(2, -1)$

Exercise 18* (Revision)

- Find the gradients of the lines parallel to AB when
 - A is $(-2, -1)$, B is $(4, 2)$
 - A is $(-1, 4)$, B is $(1, -1)$
- The Leaning Tower of Pisa is 55 m high, and the gradient of its lean is 11. By how much does the top overhang the bottom?
- Sketch the following graphs
 - $y = 3x - 2$
 - $y = 3 - 2x$
 - $2y = 5 - x$
 - $5x + 3y = 10$
- Find b such that the line from the origin to $(3, 4b)$ is parallel to the line from the origin to $(b, 3)$.
- Find the equation of the lines passing through $(6, 4)$ that are parallel and perpendicular to $3y = x + 21$.
- A temperature F in degrees Fahrenheit is related to the temperature C in degrees Celsius by the formula $F = \frac{9}{5}C + 32$.
 - Draw a graph of F against C for $-50 \leq C \leq 40$.
 - Use your graph to estimate 80°F and -22°F in degrees Celsius and 25°C in degrees Fahrenheit.
 - Use your graph to find which temperature has the same value in both degrees Fahrenheit and degrees Celsius.



Write down the equations that will produce these patterns.



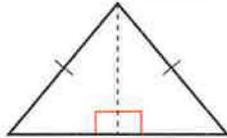


Basic principles

Triangles

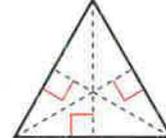
(A dashed line indicates an axis of symmetry.)

Isosceles triangle



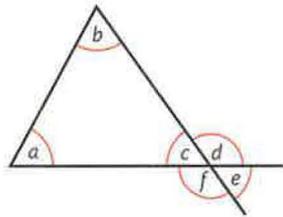
Acute, obtuse and right angles are possible.

Equilateral triangle



The rotational symmetry is of order 3.

Angle properties

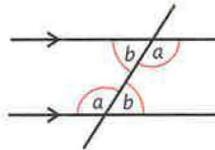


- $a + b + c = 180^\circ$ (Angle sum of triangle)
- $c + d = 180^\circ$ (Angles on straight line are supplementary)
- $c = e$ (Vertically opposite angles)
- $c + d + e + f = 360^\circ$ (Angles at a point)
- Since $d = 180^\circ - c$ (Angles on straight line)
- and $a + b = 180^\circ - c$ (Angle sum of triangle)
- $d = a + b$ (Exterior angle of triangle)

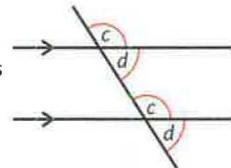
Parallel lines

Alternate angles are equal.

Co-interior angles
 $a + b = 180^\circ$

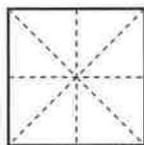


Corresponding angles are equal.



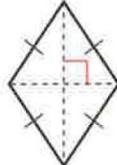
Quadrilaterals

Square



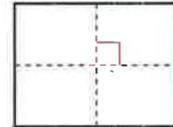
Rotational symmetry of order 4

Rhombus



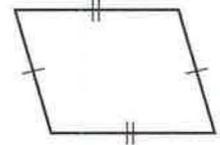
Rotational symmetry of order 2

Rectangle



Rotational symmetry of order 2

Parallelogram

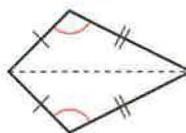


Rotational symmetry of order 2

Arrowhead

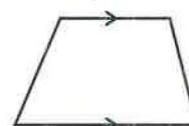


Kite



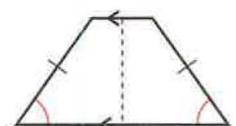
Acute, obtuse and right angles are possible.

Trapezium



There is no symmetry. Right angles are possible.

Isosceles trapezium



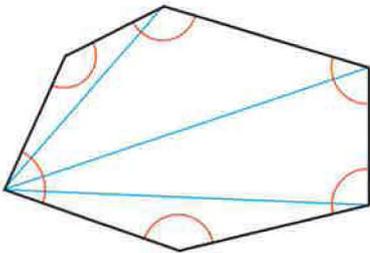
Activity 6

Copy and complete this table to show which properties are true for each type of quadrilateral.

Property	Square	Rectangle	Rhombus	Parallelogram	Arrowhead	Trapezium	Kite
The diagonals are equal in length.				No			
The diagonals bisect each other.				Yes			
The diagonals are perpendicular.				No			
The diagonals bisect the angles at the corners.				No			
Both pairs of opposite angles are equal.				Yes			

Angles of a regular polygon

Interior angles of an n -sided polygon



The polygon can be divided into $(n - 2)$ triangles.
Therefore the angle sum = $(n - 2) \times 180^\circ$.

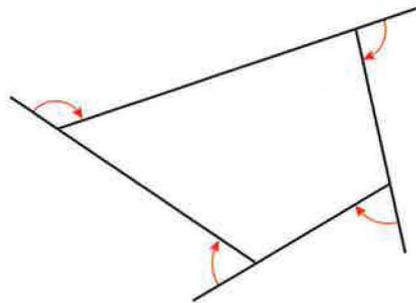
Interior and exterior angles add up together to $180n^\circ$.

In a *regular* polygon, all the interior angles are equal and all the exterior angles are equal.

$$\text{Each interior angle} = \frac{(n - 2) \times 180^\circ}{n}$$

$$\text{Each exterior angle} = \frac{360^\circ}{n}$$

Exterior angles of an n -sided polygon



The angles add up to one complete turn.
Therefore they sum to 360° .

Example 1

Find the angle sum of a polygon with seven sides.

$$n = 7$$

$$\begin{aligned} \text{Angle sum} &= (7 - 2) \times 180^\circ \\ &= 5 \times 180^\circ = 900^\circ \end{aligned}$$

Example 2

A regular polygon has ten sides. Find the size of each interior and each exterior angle.

$$n = 10$$

$$\begin{aligned} \text{Interior angle} &= \frac{(10 - 2) \times 180^\circ}{10} \\ &= 8 \times 18^\circ = 144^\circ \end{aligned}$$

$$\text{Exterior angle} = 180^\circ - 144^\circ = 36^\circ$$

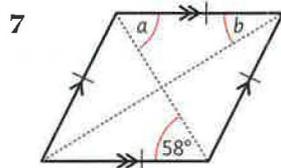
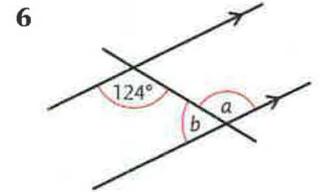
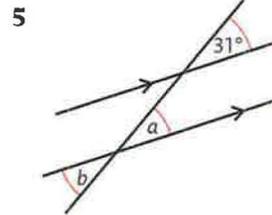
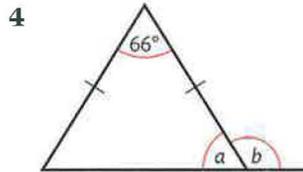
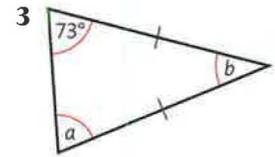
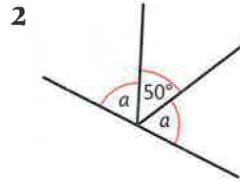
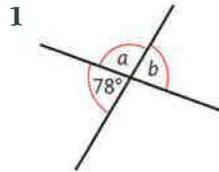
Or, find the exterior angle first.

$$\text{Exterior angle} = \frac{360^\circ}{10} = 36^\circ$$

$$\text{Interior angle} = 180^\circ - 36^\circ = 144^\circ$$

Exercise 19

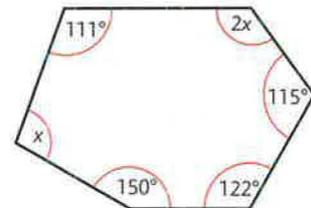
Calculate the size of each lettered angle.



- 8 A regular polygon has eight sides.
 a Calculate the size of the exterior angles.
 b Calculate the size of the interior angles.
 c Calculate the sum of the interior angles.

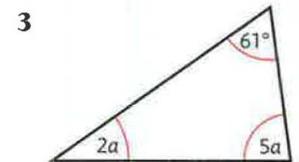
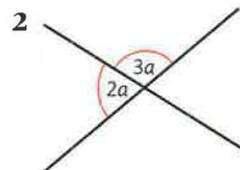
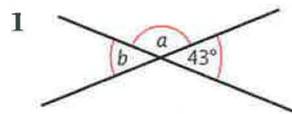
9 The angle sum of an irregular polygon is 1260° . How many sides has it?

10 Calculate the size of the two unknown angles.

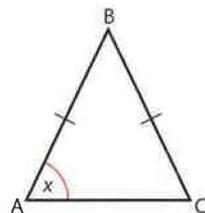


Exercise 19*

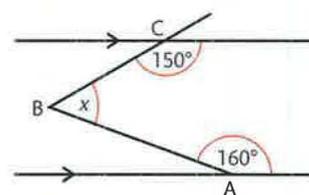
Calculate the size of each lettered angle.



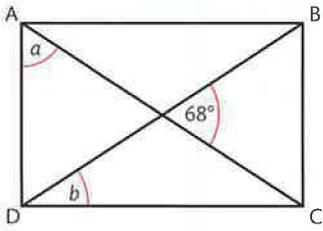
4 Express $\angle ABC$ in terms of x .



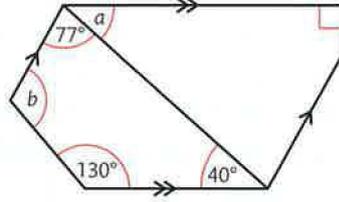
5 Find the size of angle x .
 Write out a 'solution' giving a reason for each step.



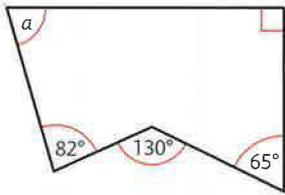
- 6 ABCD is a rectangle. Find angles a and b , giving reasons with each step.



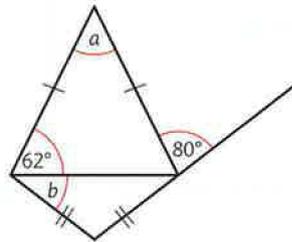
- 7 Find angles a and b , giving reasons with each step.



- 8 Find, giving reasons, the size of angle a .



- 9 Find angles a and b , giving reasons with each step.



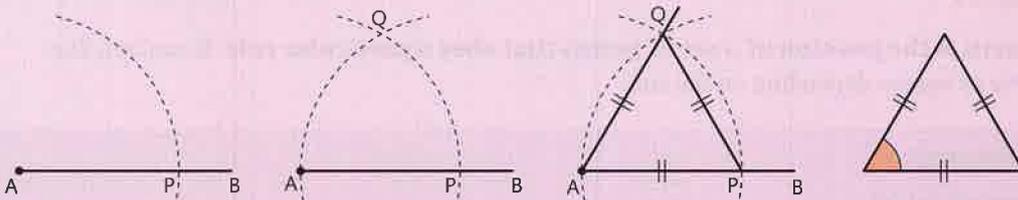
Constructions

The properties of triangles and quadrilaterals are used in the standard ruler and compass constructions.

Remember

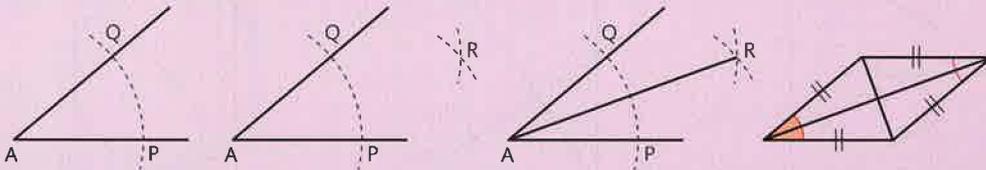
Constructing a 60° angle (equilateral triangle)

- Draw an arc from A to intersect AB at P.
- With the same radius, draw an arc from P to intersect the first arc at Q.
- Draw the line AQ. $\angle BAQ = 60^\circ$.



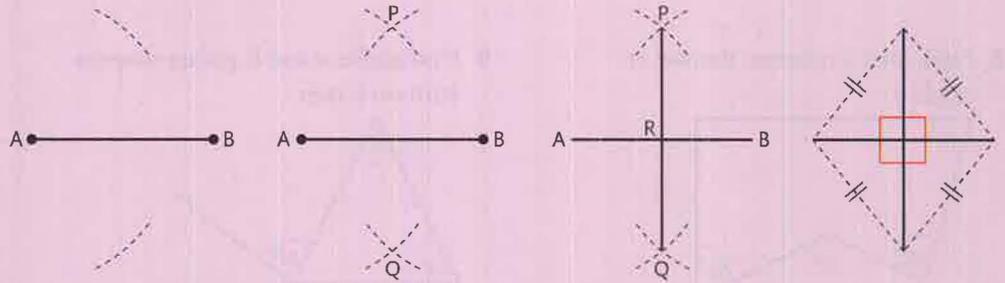
Bisecting an angle (diagonals of a rhombus or kite)

- Draw an arc from A to intersect the lines at P and Q.
- Draw arcs, with the same radius, from P and Q to intersect each other at R.
- Draw AR. $\angle PAR = \angle QAR$.



Remember**Constructing a perpendicular bisector of a line (diagonals of a rhombus)**

- Draw arcs from A, with the same radius, above and below the line.
- With the same radius, draw arcs from B to intersect those from A, above and below the line. Label these two intersections P and Q.
- Draw the line PQ. **PQ is the perpendicular bisector of AB.**
(Note: R is the mid-point of AB.)

**Remember****Constructing a perpendicular from a point X on the line**

- With the same radius, draw arcs from the point X to cut the line at either side. Label these points A and B.
- The perpendicular bisector of this part AB of the line will pass through X.

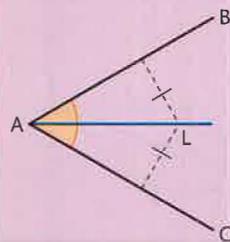


Loci

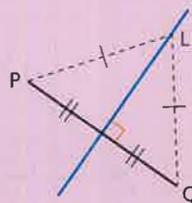
A **locus** is the position of a set of points that obey a particular rule. It can be a line, curve or region, depending on the rule.

Remember**Common loci**

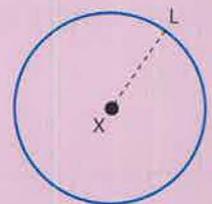
Points on the **angle bisector** of $\angle BAC$ are equidistant from the lines AB and AC.



Points on the **perpendicular bisector** of PQ are equidistant from P and Q.



Points on the **circle** with centre at X are equidistant from X.



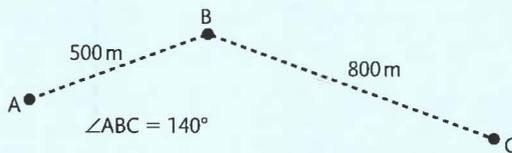
Example 3

A 10 km road race attracts so many runners that it is decided to split up the beginning of the race and have three different starts in a park.

The top diagram shows the three starting positions, A, B and C.

On an accurate scale drawing, show the point P where the three routes must converge so that they are all of the same distance.

Measure AP, and hence calculate the distance from each starting position to P.

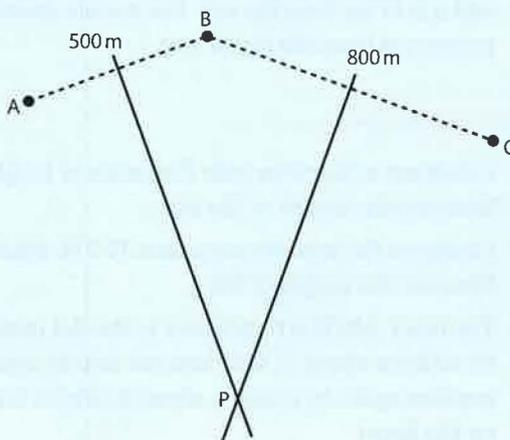


Choose a scale 1 cm : 50 m. As P is to be equidistant from A and B, it must be on the perpendicular bisector of AB. Draw this.

As P is also to be equidistant from B and C, it must be on the perpendicular bisector of BC. Draw this.

The point of intersection is equidistant from A and B and C.

AP = 19 cm. Therefore the distance from each starting position to P is 950 m.



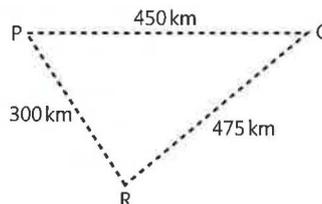
Exercise 20

Questions 1–6 should be done on plain paper. Protractors can be used, and all construction arcs are to be shown. Make a rough sketch of the figure before you begin a construction.

- Construct triangle ABC, where $AB = 8$ cm, $\angle A = 60^\circ$, and $\angle B = 45^\circ$. Measure the length of AC.

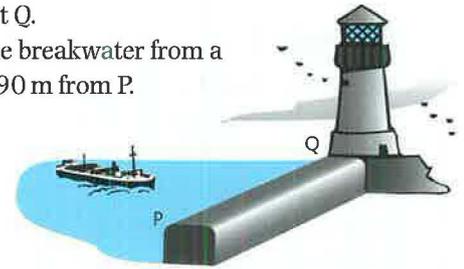
- A fierce dog is tethered by a rope 10 m long to a post 6 m from a straight path. If the path is 2 m wide, draw a scale diagram to illustrate the area of path along which a walker would be in danger.

- P, Q and R represent the positions of three radio beacons. Signals from P have a range of 300 km, Q has a range of 350 km and R has a range of 200 km.
 - Reproduce the diagram and shade the region in which all three signals can be received.
 - Measure the shortest distance from Q to this region.



- Gas rig Beta is 7 km from gas rig Gamma on a bearing of 210° . Bearings are measured from North in a clockwise direction. The region less than 4 km from gas rig Beta is an exclusion zone for ships.
 - Using a scale of 1 cm to 1 km, draw a scale diagram showing the positions of the gas rigs, and shade the region that represents the exclusion zone.
 - A boat sails so that it is always the same distance from Gamma and Beta. Draw the route taken by the boat.
 - For what distance is the boat within 4 km of oil rig Beta?

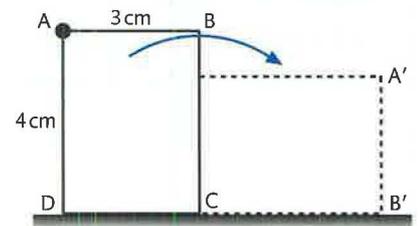
- 5 PQ is a breakwater, 750 m long, with a lighthouse at Q. Using a scale drawing, find the closest distance to the breakwater from a ship which is 280 m from the lighthouse at Q and 190 m from P.



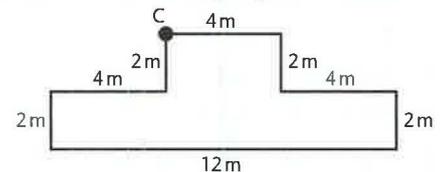
- 6 Some treasure is hidden in a field in which there are three trees: an ash A, a beech B and a chestnut C. $BC = 300$ m, $CA = 210$ m and $AB = 165$ m. The treasure is the same distance from the chestnut as from the beech, and it is 60 m from the ash. Use a scale drawing to find out how far the treasure is from the beech tree.

Exercise 20*

- Construct a rhombus with diagonals of length 9 cm and 6 cm. Measure the length of the side.
- Construct the isosceles trapezium TUVW, where $TU = 8.5$ cm, $VW = 5$ cm and $\angle UTW = 75^\circ$. Measure the length of TW.
- The block ABCD is tipped over to the flat position by rotation about C. It is then put into an upright position again by rotation about B (which is then on the floor). Draw a horizontal line to represent the floor, and then draw the locus of A during these two movements.



- 4 The diagram shows a sheep pen that is in the middle of a field. A sheepdog is tethered at the corner C by a rope 6 m long.



- Draw a scale diagram of the pen, and shade the region that the dog can cover if he is outside the pen.
 - Shade the region that he can cover if he is inside the pen.
- 5 A ladder is 15 m long. It is resting almost vertically against a wall. The bottom of the ladder is pulled out from the wall and allowed to slide into the horizontal position. Draw x - and y -axes from 0 to 15, and make a scale drawing of the locus of the middle rung of the ladder. (A 15 cm ruler may be useful.)

Exercise 21 (Revision)

Use a compass and ruler to draw the following. Remember to show all your construction arcs.

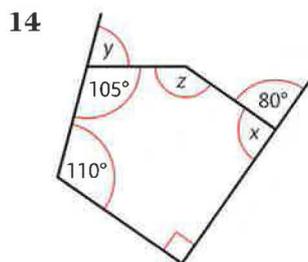
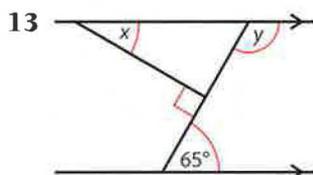
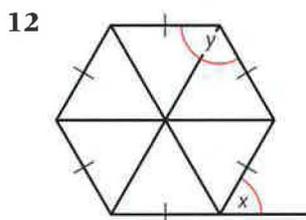
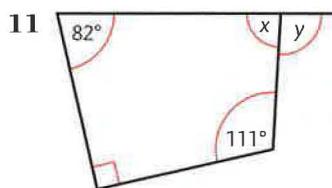
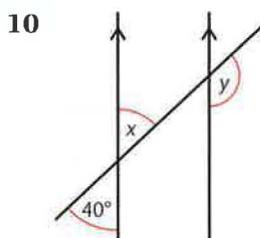
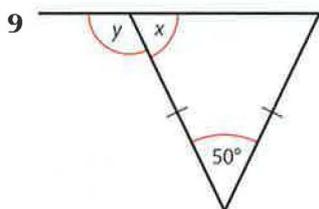
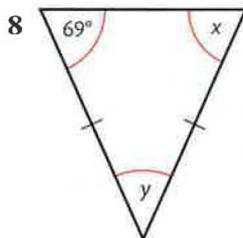
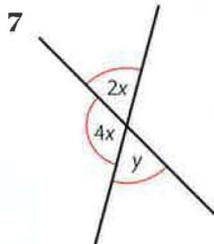
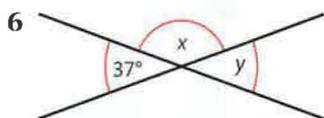
- An equilateral triangle of sides 7 cm.
- A triangle of sides 7 cm, 8 cm and 9 cm.
- The perpendicular bisector of the line AB where $AB = 8$ cm.
- Angles of 30° , 60° and 45° .

5 In a party game, a valuable prize is hidden within a triangle formed by an Oak tree (O), an Apple tree (A) and a Plum tree (P).

a Given that $OA = 16$ m, $AP = 18$ m and $OP = 20$ m construct the triangle OAP using a scale of $1 \text{ cm} = 2$ m.

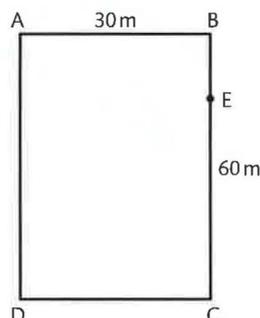
b The prize is equidistant from the Apple tree and the Plum tree and 12 m from the Oak tree. By careful construction find the distance of the prize from the Plum tree.

Calculate the size of each lettered angle.

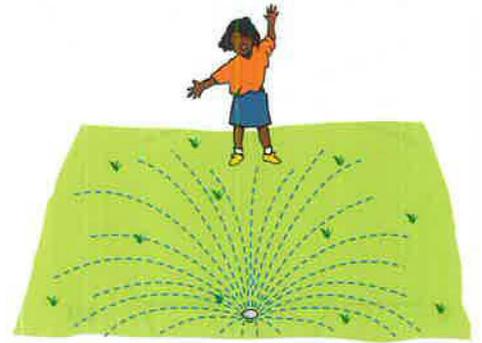
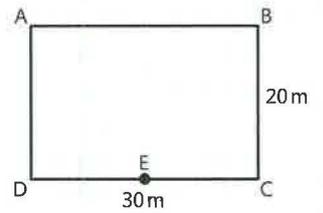


Exercise 21* (Revision)

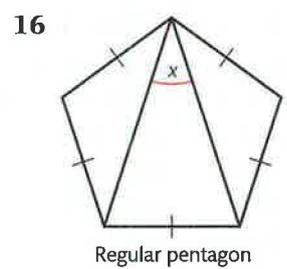
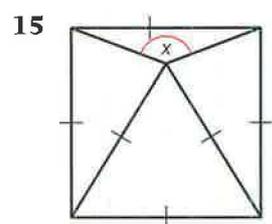
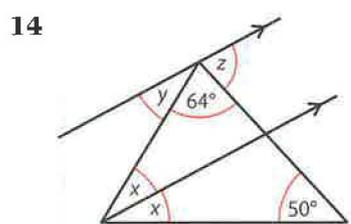
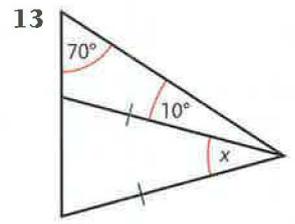
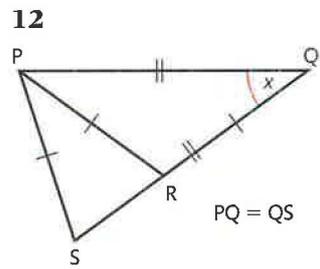
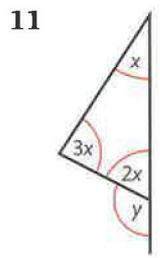
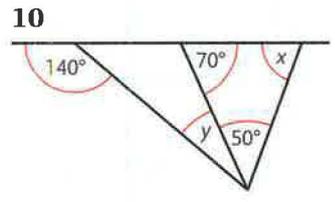
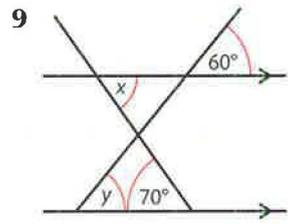
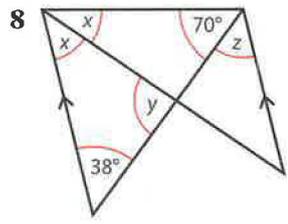
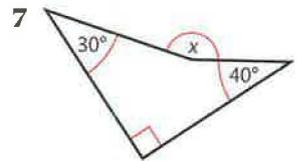
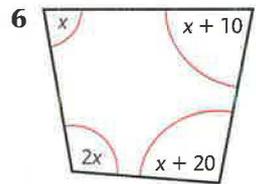
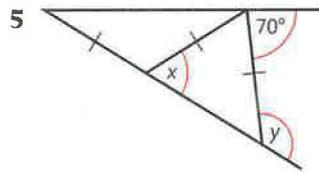
- A regular decagon has ten sides. Calculate
 - the size of the exterior angles.
 - the size of the interior angles.
 - the sum of the interior angles.
- The angle sum of a regular polygon is 3600° . Calculate
 - the number of sides of this polygon.
 - the exterior angle of this polygon.
- The rectangle ABCD represents a map of an area $30 \text{ m} \times 60 \text{ m}$. A mobile phone mast, M, is to be placed such that it is equidistant from A and B and 20 m from point E, such that $BE : EC = 1 : 2$.
 - Draw the map using a scale drawing of $1 \text{ cm} = 5 \text{ m}$.
 - Showing your construction lines clearly, find the shortest distance of M from D.



- 4 The diagram represents a rectangular lawn. There is a water sprinkler at the point E, halfway between C and D. The sprinkler wets the area within 15 m from E.
- Using a scale of 1 cm to 5 m, draw a diagram of the garden, and shade the area wetted by the sprinkler.
 - A child is playing on the lawn. She starts at A, and then runs across the lawn, keeping the same distance from the sides AD and AB until she is 10 m from the side DC. She then runs straight to the corner B. Draw the path that the child takes onto your diagram.
 - What length of her path is wet?



Calculate the size of each lettered angle.



Basic ideas of sets

The concept of a set is a simple but powerful idea. The theory of sets is mainly due to the work of the German mathematician, Cantor. It led to arguments and controversy, but by the 1920s his ideas were generally accepted and led to great advances in mathematics.

The objects can be numbers, animals, ideas, colours, in fact anything you can imagine.

A set can be described by **listing** all the members of the set, or by giving a **rule** to describe the members. The list or rule is enclosed by **braces** $\{ \}$.

Example 1

A set described by a list:

$\{\text{Maureen, John, Louice}\}$ is the set consisting of the three people called Maureen, John and Louice.

Example 2

A set described by a rule:

$\{\text{even numbers between 1 and 11}\}$ is the set consisting of the five numbers 2, 4, 6, 8, 10.

Sets are often labelled by a single capital letter. $A = \{\text{odd numbers between 2 and 10}\}$ means A is the set consisting of the four numbers 3, 5, 7, 9.

Sets can be infinite in size, for example the set of prime numbers.

Example 3

If $E = \{2, 8, 4, 6, 10\}$ and $F = \{\text{even numbers between 1 and 11}\}$, then:

$n(E) = 5, n(F) = 5$; in other words both E and F have the same number of elements

$3 \notin E$ means 3 is not a member of the set E

$6 \in F$ means 6 is a member of the set F

$E = F$ because both E and F have the same members. The order in which the members are listed does not matter.

The concept of the empty set might seem strange, but it is very useful.

Example 4

Give two examples of the empty set.

- a The set of people you know over 4 m tall.
- b The set of odd numbers divisible by two.

Key Point

A **set** is a collection of objects, which are called the elements or members of the set.

Key Point

The **number of elements** in the set A is written as $n(A)$.

Key Point

Membership of a set is indicated by the symbol \in and non-membership by the symbol \notin .

Key Point

The **empty set** is the set with no members. It is denoted by the symbol \emptyset or $\{ \}$.

Exercise 22

1 Write down two more members of each of these sets.

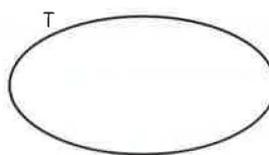
a $\{\text{carrot, potato, pea, ...}\}$

b $\{\text{red, green, blue, ...}\}$

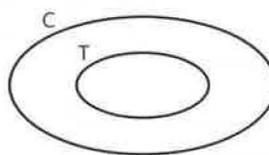
c $\{a, b, c, d, ...\}$

d $\{1, 3, 5, 7, ...\}$

When the number of elements in a set is so large that they cannot all be shown, then a simple closed curve is drawn to indicate the set. If $T = \{\text{all tabby cats}\}$ then this is shown on a Venn diagram as



If $C = \{\text{all cats in the world}\}$, then T and C can be shown on a Venn diagram as



The set T is shown inside the set C because every member of T is also a member of C .

Key Point
If T is inside C , then T is called a **subset** of C . This is written as $T \subset C$.

Example 5

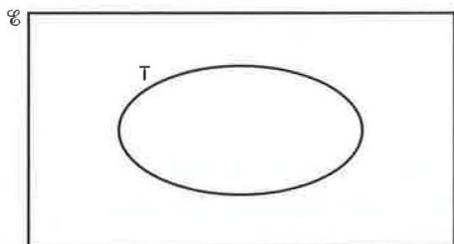
$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

- a** List the subset $O = \{\text{odd numbers}\}$.
- b** List the subset $P = \{\text{prime numbers}\}$.
- c** Is $Q = \{8, 4, 6\}$ a subset of A ?
- d** Is $R = \{0, 1, 2, 3\}$ a subset of A ?

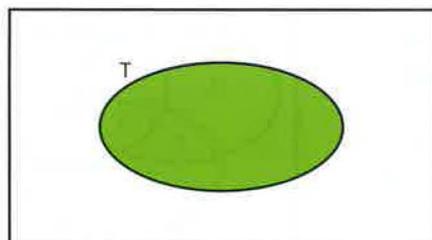
Answers

- a** $O = \{1, 3, 5, 7, 9\}$
- b** $P = \{2, 3, 5, 7\}$
- c** Q is a subset of A ($Q \subset A$) because every member of Q is also a member of A .
- d** R is not a subset of A ($R \not\subset A$) because the element 0 is a member of R but is not a member of A .

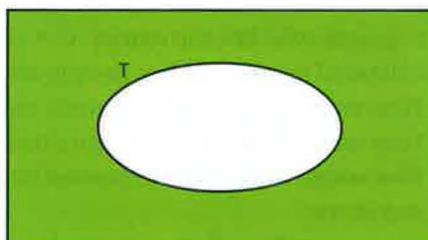
If the problem was only about cats in this world and wasn't concerned about cats outside this world, then it is more usual to call the set C the **universal set**, denoted by \mathcal{U} . The universal set contains all the elements being discussed in a particular problem, and is shown as a rectangle, where $T = \{\text{all tabby cats}\}$.



If the problem was only about cats in Rome then $\mathcal{U} = \{\text{all cats in Rome}\}$; the Venn diagram does not change. The cats outside T are all non-tabby cats. This set is denoted by T' and is known as the **complement** of T .

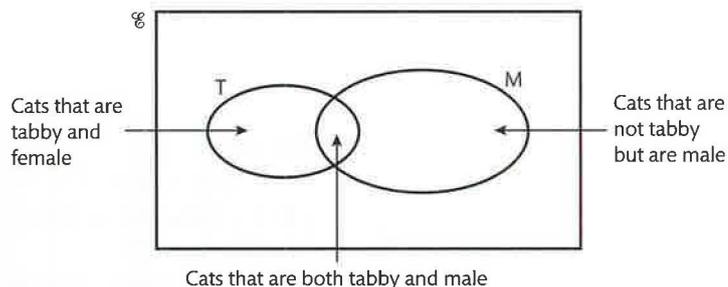


T shown shaded



T' shown shaded

Intersection and union



Sets can overlap. Let $M = \{\text{all male cats}\}$. T and M overlap because some cats are both tabby and male. T and M are shown on this Venn diagram:

The set of cats that are both tabby and male is where the sets T and M overlap.

Key Point

Where T and M overlap is called the **intersection** of the two sets T and M , and is written $T \cap M$.

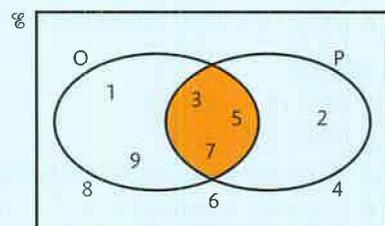
Example 6

$\mathcal{U} = \{\text{all positive integers less than 10}\}$, $P = \{\text{prime numbers less than 10}\}$ and $O = \{\text{odd numbers less than 10}\}$.

- Illustrate these sets on a Venn diagram.
- Find the set $P \cap O$ and $n(P \cap O)$.
- List P' .

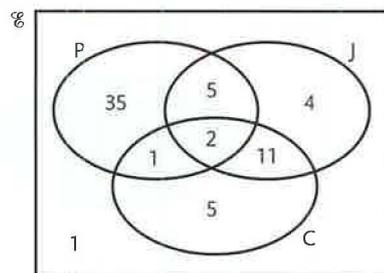
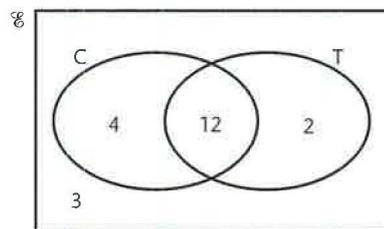
Answers

- The set $P \cap O$ is shown shaded on the Venn diagram.
- From the Venn diagram, $P \cap O = \{3, 5, 7\}$ and $n(P \cap O) = 3$.
- P' is every element not in P , so $P' = \{1, 4, 6, 8, 9\}$.

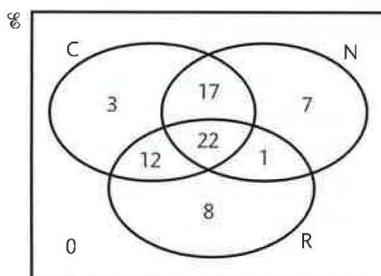


Exercise 23

- On the Venn diagram, $\mathcal{U} = \{\text{pupils in a class}\}$, $C = \{\text{pupils who like chocolate}\}$ and $T = \{\text{pupils who like toffee}\}$.
 - How many pupils like chocolate?
 - Find $n(T)$ and express what this means in words.
 - Find $n(C \cap T)$ and express what this means in words.
 - How many pupils are there in the class?
- $\mathcal{U} = \{\text{all cars in the world}\}$, $P = \{\text{pink cars}\}$, $R = \{\text{Rolls-Royce cars}\}$.
 - Describe the set $P \cap R$ in words.
 - If $P \cap R = \emptyset$, describe what this means.
- On the Venn diagram, $\mathcal{U} = \{\text{people at a disco}\}$, $P = \{\text{people who like pop music}\}$, $C = \{\text{people who like classical music}\}$ and $J = \{\text{people who like jazz}\}$.
 - How many people liked pop music only?
 - How many liked pop music and classical music?
 - How many liked jazz and classical music, but not pop music?
 - How many liked all three types of music?
 - How many people were at the disco?



- 4 On the Venn diagram, $\mathcal{U} = \{\text{ice-creams in a shop}\}$,
 $C = \{\text{ice-creams containing chocolate}\}$,
 $N = \{\text{ice-creams containing nuts}\}$ and
 $R = \{\text{ice-creams containing raisins}\}$.
- How many ice-creams contain both chocolate and nuts?
 - How many ice-creams contain all three ingredients?
 - How many ice-creams contain just raisins?
 - How many ice-creams contain chocolate and raisins but not nuts?
 - How many different types of ice-creams are there in the shop?



Exercise 23*

- $\mathcal{U} = \{\text{all positive integers less than 12}\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 5, 6, 7, 8\}$.
 - Illustrate this information on a Venn diagram.
 - List $A \cap B$ and find $n(A \cap B)$.
 - Does $A \cap B = B \cap A$?
 - List $(A \cap B)'$.
 - Is $A \cap B$ a subset of A ?
- $\mathcal{U} = \{\text{all positive integers less than 12}\}$, $E = \{1, 2, 3, 4\}$, $F = \{5, 6, 7, 8\}$.
 - Illustrate this information on a Venn diagram.
 - List $E \cap F$.
 - If $E \cap F = \emptyset$, what does this imply about the sets E and F ?
- $\mathcal{U} = \{\text{letters of the alphabet}\}$, $V = \{\text{vowels}\}$, $A = \{a, b, c, d, e\}$, $B = \{d, e, u\}$.
 - Illustrate this information on a Venn diagram.
 - List the sets $V \cap A$, $V \cap B'$, $A' \cap B$.
 - List the set $V \cap A \cap B$.
- $\mathcal{U} = \{\text{all positive integers}\}$, $F = \{4, 8, 12, 16, 20, 24\}$, $S = \{6, 12, 18, 24\}$.
 - Illustrate this information on a Venn diagram.
 - List $F \cap S$.
 - What is the smallest member of $F \cap S$?
 - F is the set of the multiples of 4, S is the set of the multiples of 6. What is the LCM of 4 and 6? How is this related to the set $F \cap S$?
 - Use this method to find the LCM of (i) 6 and 8 (ii) 8 and 10.
- Show that a set of three elements has eight subsets, including \emptyset . Find a rule giving the number of subsets (including \emptyset) for a set of n elements.

The union of two sets is the set of elements that belong to A or to B or to both A and B .

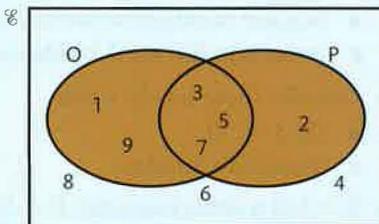
Example 7

$\mathcal{U} = \{\text{all positive integers less than 10}\}$,
 $P = \{\text{prime numbers less than 10}\}$ and
 $O = \{\text{odd numbers less than 10}\}$.

- Illustrate these sets on a Venn diagram.
- Find the set $P \cup O$ and $n(P \cup O)$.

Answers

- The set $P \cup O$ is shown shaded in the Venn diagram.
- From the Venn diagram, $P \cup O = \{1, 2, 3, 5, 7, 9\}$ and $n(P \cup O) = 6$.



Key Point

The **union** of two sets A and B is the set of elements that belong to both sets, and is written $A \cup B$.



Exercise 24

- $\mathcal{U} = \{\text{all positive integers less than } 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 4, 5, 6\}$.
 - Illustrate this information on a Venn diagram.
 - List $A \cup B$ and find $n(A \cup B)$.
 - Does $A \cup B = B \cup A$?
 - List $(A \cup B)'$.
 - Is $A \cup B$ a subset of A ?
- $\mathcal{U} = \{\text{pack of } 52 \text{ playing cards}\}$, $B = \{\text{black cards}\}$, $C = \{\text{clubs}\}$, $K = \{\text{kings}\}$.
 - Draw a Venn diagram to show the sets B , C and K .
 - Describe the set $B \cup K$.
 - Describe the set $B \cup K \cup C$.
 - Describe the set $B' \cup K$.
- $\mathcal{U} = \{\text{all triangles}\}$, $E = \{\text{equilateral triangles}\}$, $I = \{\text{isosceles triangles}\}$ and $R = \{\text{right-angled triangles}\}$.
 - Draw a Venn diagram to show the sets E , I and R .
 - Sketch a member of $I \cap R$.
 - Describe the sets $I \cup E$ and $I \cup R$.
 - Describe the sets $I \cap E$ and $E \cap R$.

Exercise 24*

- $\mathcal{U} = \{\text{all positive integers less than } 10\}$, $E = \{2, 4, 6, 8\}$, $O = \{1, 3, 5, 7, 9\}$.
 - Illustrate this information on a Venn diagram.
 - List $E \cup O$.
 - If $n(E) + n(O) = n(E \cup O)$, what does this imply about the sets E and O ?
 - If $(E \cup O)' = \emptyset$, what does this tell you about E and O ?
- In Joe's Pizza Parlour, H is the set of pizzas containing ham and C is the set of pizzas containing cheese.
 - Describe the set $H \cup C$ in words.
 - Describe the set $H \cap C$ in words.
 - If $(H \cup C)' = \emptyset$, what can you say?
- If $n(A) = n(A \cup B)$, what can you say about the sets A and B ?

Exercise 25 (Revision)

- Write down two more members of these sets.

a {salt, pepper, thyme, ...}	b {cat, dog, rabbit, ...}
c {apple, banana, orange, ...}	d {red, black, blue, ...}
- List these sets.

a {square numbers between 2 and 30}	b {all factors of 24}
c {vowels in the word 'mathematics'}	d {months of the year containing 30 days}
- Describe these sets by a rule.

a {2, 3, 5, 7}	b {32, 34, 36, 38}
c {Saturday, Sunday}	d {a, e, i, o, u}
- $\mathcal{U} = \{\text{all positive integers}\}$, $P = \{\text{prime numbers}\}$, $E = \{\text{even numbers}\}$, $O = \{\text{odd numbers}\}$.
 Say which of these are true or false.

a $51 \in P$	b P is a subset of O
c $E \cap O = \emptyset$	d $E \cup O = \mathcal{U}$

- 5 $\mathcal{E} = \{\text{positive integers less than 11}\}$
 $A = \{\text{multiples of 2}\}$ $B = \{\text{multiples of 4}\}$
 a Illustrate this information on a Venn diagram.
 b List the set A' and describe it in words.
 c What is $n(B')$?
 d Is $B \subset A$? Explain your answer.
- 6 $\mathcal{E} = \{\text{positive integers less than 15}\}$, $A = \{5, 7, 11, 13\}$, $B = \{6, 7, 9\}$, $C = \{\text{multiples of 3}\}$.
 a List C.
 b Draw a Venn diagram to illustrate the sets \mathcal{E} , A, B and C.
 c List $A \cup B$.
 d List $B \cap C$.
 e What is $A \cap C$?
- 7 Draw Venn diagrams to illustrate these statements.
 a $A \cap B = \emptyset$ b $A \cap B \neq \emptyset$
 c $A \cap B = A$ d $A \cup B = A$
- 8 $\mathcal{E} = \{\text{members of an expedition to the South Pole}\}$, $A = \{\text{people born in Africa}\}$,
 $F = \{\text{females}\}$, $C = \{\text{people born in China}\}$.
 a Describe $A \cap F$.
 b What is $A \cap C$?
 c $\text{Amber} \in A \cup C$. What can you say about Amber?
 d Illustrate the sets \mathcal{E} , A, F and C on a Venn diagram.
- 9 $\mathcal{E} = \{\text{Suzy's clothes}\}$, $D = \{\text{Dresses}\}$, $R = \{\text{Red clothes}\}$ and $G = \{\text{Green clothes}\}$
 a $D \cap R = \emptyset$. Describe what this means in words.
 b $D \subset G$. Describe what this means in words.
 c Illustrate all this information on a Venn diagram.
- 10 The following information was obtained about all the fast food restaurants in a town.
 Six sold burgers and pizzas, four sold pizzas only, nine sold burgers, while two served
 neither burgers nor pizzas.
 a Draw a Venn diagram to represent all of this information
 b How many fast food restaurants are there in the town?

Exercise 25* (Revision)

- 1 List these sets.
 a $\{\text{multiples of 4 less than 20}\}$
 b $\{\text{colours of the rainbow}\}$
 c $\{\text{arrangements of the letters CAT}\}$
 d $\{\text{all pairs of products of 1, 2, 3}\}$
- 2 Describe these sets by a rule.
 a $\{1, 2, 3, 4, 6, 12\}$
 b $\{1, 1, 2, 3, 5\}$
 c $\{\text{hearts, clubs, diamonds, spades}\}$
 d $\{\text{tetrahedron, cube, octahedron, dodecahedron, icosahedron}\}$
- 3 a A and B are two sets. A contains 12 members, B contains 17 members and $A \cup B$
 contains 26 members. How many members of A are not in $A \cap B$?
 b Draw a Venn diagram with circles representing three sets A, B and C, such that these
 are true:
 $A \cap C = \emptyset$, $B \cap C \neq \emptyset$, $A \cap B = \emptyset$.

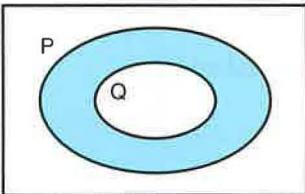


- 4 $\mathcal{C} = \{\text{pack of 52 playing cards}\}$, $A = \{\text{aces}\}$, $B = \{\text{black cards}\}$, $D = \{\text{diamonds}\}$.
- Describe $A \cap D$.
 - Describe $B \cap D$.
 - Describe $A \cup D$.
 - Find $n(A \cap B)$.
 - Illustrate the sets \mathcal{C} , A , B and D on a Venn diagram.
- 5 $\mathcal{C} = \{\text{triangles}\}$, $R = \{\text{right-angled triangles}\}$, $I = \{\text{isosceles triangles}\}$, $E = \{\text{equilateral triangles}\}$.
- Describe $I \cap R$.
 - Describe $I \cap E$.
 - Describe $R \cap E$.
 - Draw a Venn diagram to illustrate the sets \mathcal{C} , R , I and E .
- 6 $\mathcal{C} = \{\text{positive integers less than 30}\}$, $P = \{\text{multiples of 4}\}$, $Q = \{\text{multiples of 5}\}$, $R = \{\text{multiples of 6}\}$.
- List $P \cap Q$.
 - $x \in P \cap R$. List the possible values of x .
 - Is it true that $Q \cap R = \emptyset$? Explain your answer.
- 7 $\mathcal{C} = \{\text{even numbers less than 15}\}$, $A = \{\text{multiples of 4}\}$
 B satisfies $A \cap B = \emptyset$ and $n(B) = 4$.
 What is $A \cup B$?
- 8 $n(\mathcal{C}) = 17$, $n(B') = 9$ and $n(A' \cap B) = 6$
- Find $n(B)$
 - Find $n(A \cap B)$
 - Draw a Venn diagram to illustrate this information.
- 9 A class of 30 students was asked to choose **at least** one option subject from list A and list B. Two students forgot to hand their forms in. Of the rest, twenty two chose list A and twenty five chose list B.
- Draw a Venn diagram to illustrate this information.
 - How many students chose both options?
- 10 There are 30 Widgets, and every Widget is a Woodle.
 There are 20 Wopets, half of which are Woodles. No Wopet is a Widget.
 Half of all Woodles are Widgets.
- Draw a Venn diagram to represent this information.
 - How many Woodles are neither Widgets nor Wopets?

Multiple choice 1

- 1 The number 437 600 in standard form correct to 3 s.f. is
A 4.375×10^5 B 43.8×10^4 C 4.38×10^5 D 4.37×10^5
- 2 If $3x + 4 = 28 + x$, the value of x is
A 8 B 16 C 12 D 26
- 3 A rectangular field is $(2x + 1)$ m long and x m wide. If the perimeter is 14 m, the area is
A 5 m^2 B 10 m^2 C 14 m^2 D 15 m^2
- 4 A radio decreases in value by 55%. If it is now valued at \$157.50, its value before the decrease was
A \$350 B \$1575 C \$3500 D \$286
- 5 The equation of the straight line with a gradient of $\frac{1}{3}$ passing through point $(0, 2)$ is
A $3y = x + 6$ B $y = 2x + \frac{1}{3}$ C $3y + x = 6$ D $y = \frac{1}{3}x$
- 6 The equation of the straight line through points $A(0, 10)$ and $B(10, 2)$ is
A $5y - 4x = 5$ B $x + y = 10$ C $y = 4x + 50$ D $5y + 4x = 50$
- 7 The internal angle of a regular octagon is
A 158° B 140° C 120° D 135°
- 8 The larger angle between the hands of a clock at 18:05 is nearest to
A 210° B 200° C 180° D 170°
- 9 In a class of 20 pupils, 14 like reggae music, 3 like classical music and 4 do not like either. The number of pupils who like both reggae and classical is
A 0 B 1 C 2 D 3

10 %



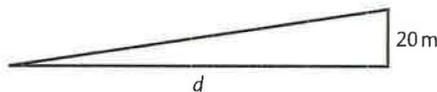
The shaded region is described by

- A Q' B $P \cup Q'$ C $P \cap Q$ D $P \cap Q'$

Self-assessment 1

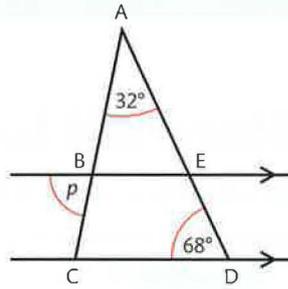


- 1 a Write the number 385 000 in standard form.
b Write the number 3.25×10^3 as an ordinary number.
- 2 a Write the number 36.5782 correct to 2 decimal places.
b Write the number 36.5782 correct to 2 significant figures.
- 3 a Calculate $(4.8 \times 10^5) \div (1.2 \times 10^3)$, giving your answer in standard form to 3 significant figures.
b Calculate $(4.8 \times 10^5) + (1.2 \times 10^3)$, giving your answer in standard form to 3 significant figures.
- 4 a Calculate $(3 \times 10^3) \times (4 \times 10^2)$, giving your answer in standard form to 3 significant figures.
b Calculate $(3 \times 10^3) - (4 \times 10^2)$, giving your answer in standard form to 3 significant figures.
- 5 a Find 30% of \$15.40.
b Decrease \$48.60 by 12%.
- 6 a Simplify the expression $3ba + 4ab - ba + 2ab$.
b Simplify the expression $4xy - x(y - 3)$.
c Simplify the expression $3ab \times a^2$.
- 7 a Solve the equation $2(3x + 1) = 20$.
b Solve the equation $2(4a - 3) - (2a + 5) = 10$.
c Solve the equation $7 - 2x = 3x - 8$.
- 8 Three consecutive numbers sum to 525.
a If the first of the consecutive numbers is x , what is the second number?
b Write down an equation in x .
c Solve your equation to find x .
- 9 Which two of the following lines are parallel?
 $x - 3y = 12$, $3y + x = 5$, $y = 3x - 2$, $6y - 2x = 7$
- 10 Find the gradient of the line through these pairs of points.
a $P = (1, 5)$ and $Q = (3, 9)$ b $C = (4, 4)$ and $D = (1, 16)$
- 11 a A road has a gradient of $\frac{1}{20}$. What is the value of d ?
b The line joining $A = (2, 5)$ to $B = (6, q)$ has a gradient of $\frac{1}{2}$. Find the value of q .

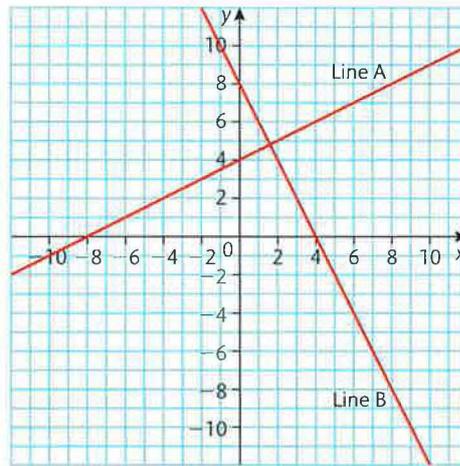


- 12 a Construct triangle ABC, with $AB = 7$ cm, angle $BAC = 75^\circ$ and angle $ABC = 60^\circ$.
b Measure AC.
- 13 a Construct triangle PQR such that $PQ = 8$ cm, $\angle PQR = 50^\circ$, and $\angle RPQ = 80^\circ$.
b Construct the perpendicular from R to intersect PQ at S.
c Measure RS, and hence calculate the area of $\triangle PQR$.

- 14 In the diagram, ABC and AED are straight lines.
BE and CD are parallel. Angle BAE = 32° and angle EDC = 68° .
Work out the value of p .

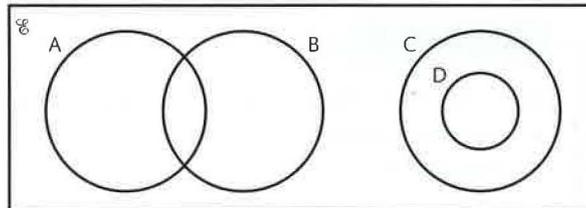


- 15 a Draw axes on graph paper with $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$.
b Draw the graph of the line $y = 3x + 2$.
c Draw the graph of the line $x + 2y = 8$.
d Write down the coordinates of the point where the two lines intersect.
- 16 Write down the gradient, intercept and equation of the lines shown on the diagram.



- 17 The Venn diagram shows four sets A, B, C, and D.

$A \subset B$	$C \cup D = D$
$A \cap B \neq \emptyset$	$A \cup C = \mathcal{E}$
$C \cup D = C$	$A \cap C = \emptyset$



Choose a statement from the box that correctly describes the relationship between

- a A and C b D and C c A and B
- 18 $n(\mathcal{E}) = 20$, $n(B') = 7$, $n(A' \cap B) = 6$
a Find $n(B)$. b Find $n(A \cap B)$.
- 19 $\mathcal{E} = \{\text{all triangles}\}$, $A = \{\text{isosceles triangles}\}$, $B = \{\text{right angled triangles}\}$
a Draw a Venn diagram to illustrate the sets A and B.
b Calculate the three angles of a member of $A \cap B$
 $C = \{\text{equilateral triangles}\}$
c Add set C to your Venn diagram.
- 20 $\mathcal{E} = \{\text{positive whole numbers less than 19}\}$, $A = \{\text{multiples of 2}\}$, $B = \{\text{multiples of 3}\}$
a List the set $A \cap B$. b Describe the set $A \cap B$.
c Describe the set A' . d List the set $A' \cap B$.
e Describe the set $A' \cap B$.

Unit 2: Number



Standard form (negative indices)

We can write numbers, however small, in **standard form**.

Key Point

$$10^{-n} = \frac{1}{10^n}$$

Example 1

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$10^{-6} = \frac{1}{10^6} = \frac{1}{1\,000\,000} = 0.000\,001$$

Activity 7

Copy and complete the table.

Decimal form	Fraction form or multiples of 10	Standard form
0.1	$\frac{1}{10} = \frac{1}{10^1}$	1×10^{-1}
	$\frac{1}{100} = \frac{1}{10^2}$	
0.001	=	
0.0001	=	
	=	1×10^{-5}

Activity 8

Write down the mass of each of the first three objects in grams

- in ordinary numbers
- in standard form.

Copy and complete these statements:

- A house mouse is ... times heavier than a pigmy shrew.
- A pigmy shrew is ... times heavier than a grain of sand.
- A grain of sand is 100 000 times lighter than a
- A pigmy shrew is 10 000 times heavier than a
- A ... is 100 million times heavier than a
- A house mouse is ... 10 000 billion times heavier than a

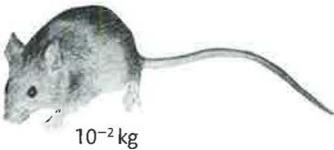
Example 2

Write 0.987 in standard form.

$$0.987 = 9.87 \times \frac{1}{10} = 9.87 \times 10^{-1}$$

To display this on your calculator, press **9 . 8 7 $\times 10^x$ (-) 1**

House mouse



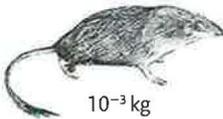
10^{-2} kg

Grain of sand



10^{-7} kg

Pigmy shrew



10^{-3} kg

Staphylococcus bacterium



10^{-15} kg

Exercise 26

For Questions 1–4, write each number in standard form.

1 0.1 2 0.001 3 $\frac{1}{1000}$ 4 10

For Questions 5–8, write each one as an ordinary number.

5 10^{-3} 6 1.2×10^{-3} 7 10^{-6} 8 4.67×10^{-2}

For Questions 9–12, write each number in standard form.

9 0.543 10 0.007 11 0.67 12 100

For Questions 13–17, write each one as an ordinary number.

Check your answers with a calculator.

13 $10^{-2} \times 10^4$ 14 $10^2 \div 10^{-2}$
 15 $(3.2 \times 10^{-2}) \times (4 \times 10^3)$ 16 $(6 \times 10^{-1}) \div (2 \times 10^{-2})$
 17 $(2 \times 10^{-2}) \times (9 \times 10^{-1})$

Exercise 26*

For Questions 1–4, write each answer as an ordinary number.

1 $10^3 \times 10^{-2}$ 2 $10^{-2} + 10^{-3}$ 3 $10^{-4} \times 10^2$ 4 $10^{-3} + 10^{-4}$

For Questions 5–8, write each answer in standard form.

5 $10 \div 10^{-2}$ 6 $10^{-1} \div 10^{-2}$ 7 $10^3 \div 10^{-1}$ 8 $10^{-2} \div 10^{-4}$

You will need this information to answer Questions 9 and 10.

Cough virus



9.144×10^{-6} mm diameter

Human hair



5×10^{-2} mm diameter

Pin



6×10^{-1} mm diameter

- 9 How many viruses, to the nearest thousand, can be placed in a straight line across the width of a human hair?
- 10 How many viruses, to the nearest thousand, can be placed in a straight line across the width of a pin?
- 11 A molecule of water is a very small thing, so small that its volume is 10^{-27} m³.
- How many molecules are there in 1 m³ of water?
If you wrote your answer in full, how many zero digits would there be?
 - If you assume that a water molecule is in the form of a cube, show that its side length is 10^{-9} m.
 - If a number of water molecules were placed touching each other in a straight line, how many would there be in a line 1 cm long?
 - The volume of a cup of tea is 200 cm³.
How many molecules of water would the cup hold?
If all these were placed end to end in a straight line, how long would the line be?
Take the circumference of the Earth to be 40 000 km.
How many times would the line of molecules go around the Earth?



Four rules of fractions

This section will give you practice in using fractions. The questions will help you to understand much of the algebra in this unit.

Addition and subtraction

A common denominator is required.

Example 3

Addition

$$\begin{aligned}\frac{3}{4} + \frac{1}{6} \\ = \frac{9}{12} + \frac{2}{12} = \frac{9+2}{12} = \frac{11}{12}\end{aligned}$$

Example 4

Subtraction

$$\begin{aligned}\frac{3}{4} - \frac{2}{5} \\ = \frac{15}{20} - \frac{8}{20} = \frac{15-8}{20} = \frac{7}{20}\end{aligned}$$

Example 5

With mixed fractions

$$\begin{aligned}3\frac{1}{3} - 1\frac{3}{4} \\ = \frac{10}{3} - \frac{7}{4} \\ = \frac{40-21}{12} = \frac{19}{12} = 1\frac{7}{12}\end{aligned}$$

Multiplication and division

Convert mixed fractions into improper fractions.

Example 6

Multiplication

$$\begin{aligned}1\frac{3}{4} \times \frac{3}{5} \\ = \frac{7}{4} \times \frac{3}{5} = \frac{7 \times 3}{4 \times 5} = \frac{21}{20} = 1\frac{1}{20}\end{aligned}$$

Example 7

Cancelling

$$\begin{aligned}1\frac{5}{9} \times 2\frac{1}{7} \\ = \frac{14}{9} \times \frac{15}{7} = \frac{14 \times 15}{9 \times 7}\end{aligned}$$

Divide top and bottom by 7, and by 3.

$$= \frac{2 \times 5}{3 \times 1} = \frac{10}{3} = 3\frac{1}{3}$$

Example 8

Division

$$\frac{3}{4} \div \frac{5}{11}$$

Turn the divisor upside down and multiply.

$$= \frac{3}{4} \times \frac{11}{5} = \frac{3 \times 11}{4 \times 5} = \frac{33}{20} = 1\frac{13}{20}$$

Example 9

Multiplying with a whole number.

$$\frac{3}{4} \times 7$$

Change the whole number into a fraction.

$$= \frac{3}{4} \times \frac{7}{1} = \frac{3 \times 7}{4 \times 1} = \frac{21}{4} = 5\frac{1}{4}$$

Example 10

Dividing into a whole number.

$$8 \div 1\frac{1}{2} = \frac{8}{1} \div \frac{3}{2} = \frac{8}{1} \times \frac{2}{3} = \frac{8 \times 2}{1 \times 3} = \frac{16}{3} = 5\frac{1}{3}$$

Exercise 27

Work these out.

- | | | | |
|----------------------------------|-------------------------------------|---------------------------------------|------------------------------------|
| 1 $\frac{2}{7} + \frac{4}{7}$ | 2 $\frac{3}{10} + \frac{1}{10}$ | 3 $\frac{7}{9} + \frac{4}{9}$ | 4 $\frac{5}{6} - \frac{1}{3}$ |
| 5 $\frac{3}{8} + \frac{7}{12}$ | 6 $3\frac{1}{4} + 1\frac{1}{6}$ | 7 $\frac{5}{6} \times \frac{1}{3}$ | 8 $\frac{3}{4} \div \frac{7}{8}$ |
| 9 $4 \times \frac{3}{20}$ | 10 $\frac{12}{25} \div 4$ | 11 $2\frac{1}{7} \times 1\frac{2}{5}$ | 12 $1\frac{1}{8} \div \frac{3}{4}$ |
| 13 $2\frac{5}{6} + 1\frac{3}{4}$ | 14 $5\frac{3}{10} - 2\frac{11}{20}$ | 15 $1\frac{3}{5} \times 3$ | 16 $1\frac{4}{5} \div 6$ |

Exercise 27*

Work these out.

- | | | | |
|-------------------------------------|-------------------------------------|---|---|
| 1 $\frac{1}{3} + \frac{5}{12}$ | 2 $\frac{5}{6} - \frac{7}{30}$ | 3 $\frac{1}{5} + \frac{3}{10} + \frac{9}{20}$ | 4 $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ |
| 5 $\frac{5}{8} \times \frac{8}{25}$ | 6 $\frac{2}{3} \div \frac{20}{21}$ | 7 $\frac{2}{3} \div 3$ | 8 $\frac{1}{2} \times \frac{15}{16} \times \frac{4}{5}$ |
| 9 $4\frac{1}{2} + 3\frac{1}{6}$ | 10 $7\frac{2}{3} - 1\frac{1}{6}$ | 11 $7\frac{2}{3} - \frac{8}{9}$ | 12 $3\frac{1}{7} \times \frac{7}{15}$ |
| 13 $4 \times \frac{2}{3}$ | 14 $2\frac{1}{3} \div 2\frac{4}{5}$ | 15 $14 \div 3\frac{1}{9}$ | 16 $(2\frac{1}{8} + 2\frac{1}{4}) \div 2\frac{1}{3}$ |

Ratio

Example 11

A marinade in a recipe contains rice wine and soy sauce in the ratio 2 : 3.

How much of each ingredient is needed for 100 ml of the mixture?

(Add the ratios together; $2 + 3 = 5$.)

Then the parts are in the ratio $\frac{2}{5} : \frac{3}{5}$.

Amount of rice wine = $\frac{2}{5}$ of 100 = 40 ml. Amount of soy sauce = $\frac{3}{5}$ of 100 = 60 ml.

Exercise 28

- 1 Divide \$392 in the ratio of 3 : 4.
- 2 Divide 752 kg in the ratio of 1 : 7.
- 3 Divide 984 in the ratio of 7 : 5.
- 4 Divide 13.5 in the ratio of 3 : 2.

Example 12

Divide £1170 in the ratio of 2 : 3 : 4. (Add the ratios together; $2 + 3 + 4 = 9$.)

Then the first part = $\frac{2}{9}$ of £1170 = £260.

The second part = $\frac{3}{9}$ of £1170 = £390.

The third part = $\frac{4}{9}$ of £1170 = £520. (Check: £260 + £390 + £520 = £1170.)

Exercise 28*

- 1 Divide \$120 in the ratio 3 : 5.
- 2 The fuel for a lawn mower is a mixture of 8 parts petrol to one part oil. How much oil is required to make 1 litre of fuel?
- 3 Mr Chan has three daughters, An, Lien and Tao, aged 7, 8 and 10 years respectively. He shares \$100 between them in the ratio of their ages. How much does Lien receive?
- 4 A breakfast cereal contains the vitamins thiamin, riboflavin and niacin in the ratio 2 : 3 : 25. A bowl of cereal contains 10 mg of these vitamins. Calculate the amount of riboflavin in a bowl of cereal.

Positive integer powers of numbers

Powers are used to write certain numbers in a convenient way. To help you understand how the rules of indices work, study the table carefully.

Operation	Example	Rule
Multiplying	$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$ $= 3^6 = 3^{4+2} = 729$	Add the indices $(a^m \times a^n = a^{m+n})$
Dividing	$3^4 \div 3^2 = \frac{3 \times 3 \times 3 \times 3}{3 \times 3} = 3^2 = 3^{4-2} = 9$	Subtract the indices $(a^m \div a^n = a^{m-n})$
Raising to a power	$(3^4)^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$ $= 3^8 = 3^{4 \times 2} = 6561$	Multiply the indices $(a^m)^n = a^{mn}$