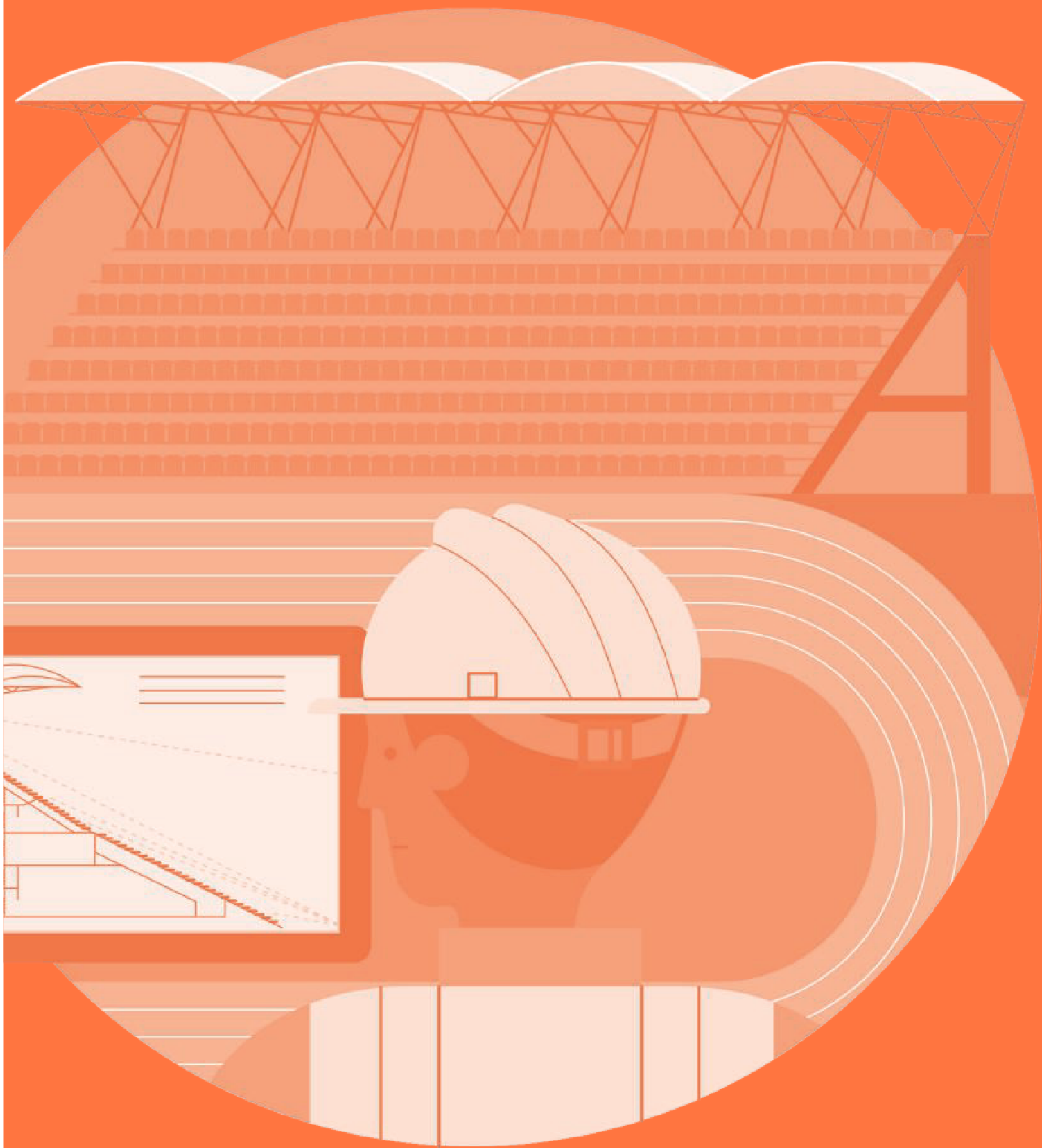
Oxford International Resources

9

Maths

Teacher's Guide



Lower Secondary

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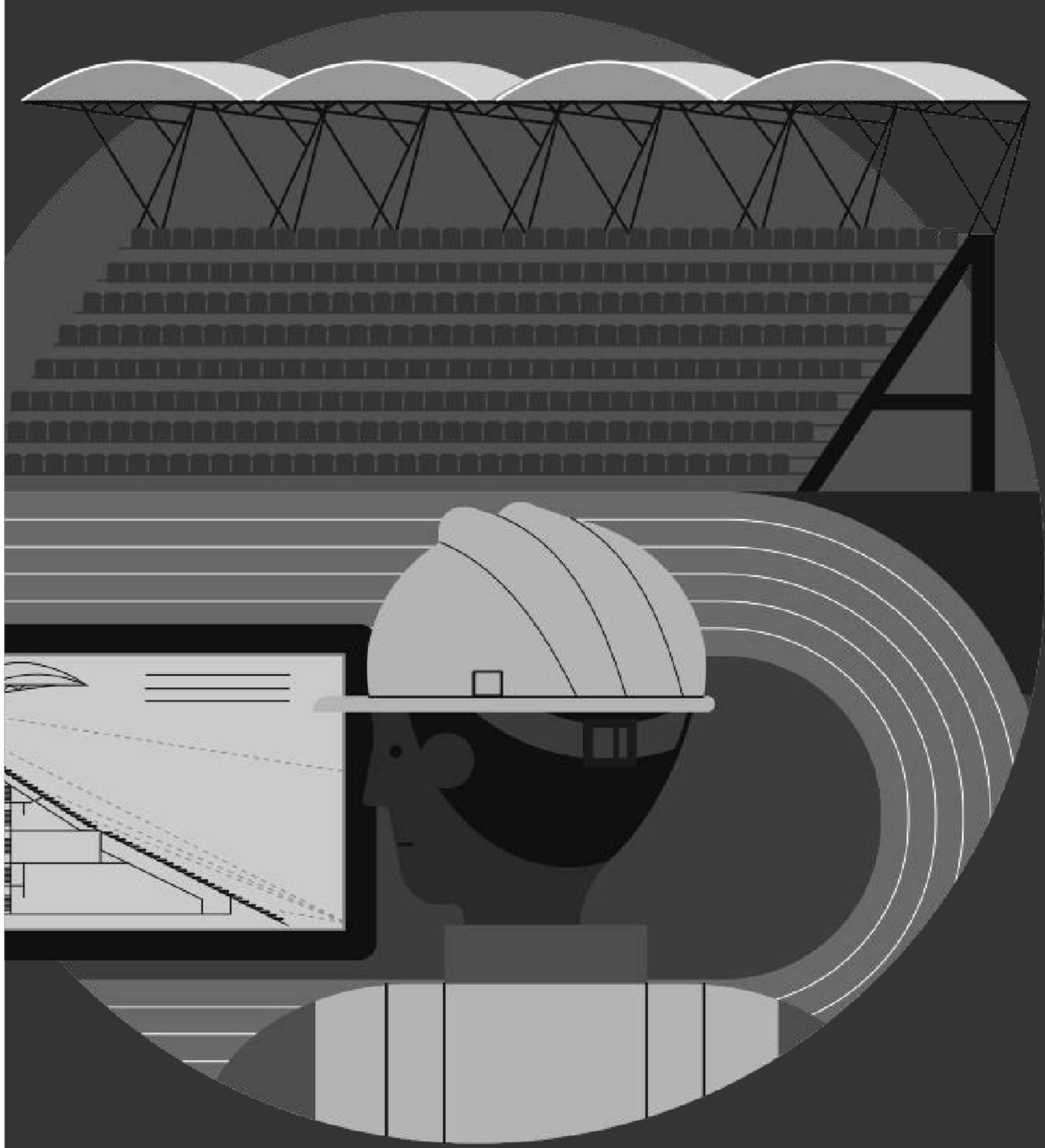

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Contents

Introduction

The joy of learning maths

We are living in an ever-changing world, where the way we work, live, learn, communicate, and relate to one another is constantly shifting. In this climate, we need to instil in our learners the skills to equip them for every eventuality so they are able to overcome challenges, adapt to change, and have the best chance of success. To do this, we need to evolve beyond traditional teaching approaches and foster an environment where students can start to build lifelong learning skills. Students need to learn how to learn, problem-solve, be agile, and work flexibly. Going hand in hand with this is the development of self-awareness and mindfulness through the promotion of wellbeing to ensure that students learn the socio-emotional skills to succeed.

Teaching and learning with *Oxford International Maths*

This course is suitable for use alongside the Oxford International Curriculum and the English National Curriculum. The books for each year (or stage) follow the scheme and meet the learning objectives for both curricula. Objectives are written in student-friendly language in the Student Book.

The Lower Secondary course is designed for students aged 11 to 14. Each year has a Student Book and a Teacher's Guide. There are also numerous digital resources and sources of support on www.kerboodle.com. These three core elements provide a cohesive offer that supports learners to develop and consolidate knowledge and skills, and to draw connections between topics.

Oxford International Maths at Lower Secondary is a mastery course. The term 'mastery' has been drawn from teaching approaches in countries where maths performance is high. The essence of mastery is to produce students who have deep conceptual understanding and procedural fluency through learning in a collaborative and problem-solving context. Mastery learning incorporates exposure to different methods of solving a problem, mathematical dialogue, and explanation.

This course has six main aims:

1 Provide clear and coherent curriculum pathways

Close attention is paid to the sequencing of concepts, connections with prior learning, common difficulties and misconceptions, and consistent use of key models and language.

Pre-requisite, current, and future learning are signposted to ensure that learners are clear where they are on their learning journey. Checks and diagnostic questions help teachers identify where learners are before teaching a new topic, and integrated assessment provides regular feedback on learners' progress against the curriculum pathway.

2 Hold high expectations, aspirations, and ambitions for all learners

Developing, secure, and extending (DSE) learning outcomes underpin each learning objective to ensure that the same curriculum is accessible to all, with the aim that all learners achieve a secure level.

Support for reactivating prior learning and identifying misconceptions at the start of each topic helps reduce the attainment gap and ensures that all learners are at a suitable starting point for the new content. Carefully graduated practice allows all learners to build confidence while also providing stretch for those who are ready for more challenge. Differentiation is supported through questions targeted at each of the DSE outcomes.

3 Support engaged, self-regulated, and metacognitive learning

The Reflect, Expect, Check, Explain (RCE) framework has been developed by Craig Barton. It encourages metacognitive thinking where students think about their learning, feel empowered to reflect on their strengths and areas for development, and make connections across topics.

4 Promote development of learner identity and identification with each subject

The relevance of maths to learners is highlighted throughout curriculum components and resources, including student-facing facts and visuals that encourage thinking about real-world applications and provide prompts for discussion.

5 Enable responsive teaching and learning that continually evolves and improves

Resources are provided to help teachers check understanding at every stage in the learning process, and guidance about common difficulties and misconceptions supports teachers to respond to learners' needs.

Regular formative assessment on your Kerboodle subscription provides learners with personalized next steps to secure or extend their learning. Detailed reporting on Kerboodle supports both teachers and learners.

6 Inspire fascination and awe and wonder in the world around us

This course aims to stimulate fascination through a strong focus on thinking and working mathematically. Opportunities to develop the skills associated with working like a mathematician are embedded throughout.

Teaching techniques

Grouping students to promote a growth mindset

It is expected that you will use a variety of student groupings. It is important that you are active in deciding which form of grouping is appropriate, depending on the activity. In this way, students will learn how to work in a variety of ways and with a range of different students.

There are three main ways of grouping students:

- **Friendship groups** are most appropriate for activities in which students have been given some element of choice, for example, if they are carrying out some research for a data handling project. This grouping is the default if teachers do not actively group students.
- **Ability groups**, or groups based on students' prior experience, may be helpful if the lesson requires a very specific prior knowledge. You can group together the students that you know have this knowledge, and they can then work with minimal guidance from you. This will allow you to focus on groups who need additional support.
- **Mixed-attainment groups** are encouraged for the majority of activities. This form of grouping is also favoured by those following a mastery approach. Working in collaborative, all-attainment groups also supports students' wellbeing and promotes a growth mindset, as described in research by Carol Dweck. She found that students who were grouped by ability tended to stay in those groupings throughout their school life, and regard themselves as having a fixed ability that could not be changed. This has dire consequences for students in middle or lower sets. When placed in mixed-ability groups, all students can develop a growth mindset that enables them to believe they can learn and improve, whatever their starting point (Dweck, C., 2007. 'The Perils and Promise of Praise'. *Educational Leadership*. October 2007, 65(2), 34–39). A growth mindset is promoted when students do not feel that their future success is predicated on prior achievement. This kind of grouping is particularly helpful for students new to English, as those who are less confident speaking the language will be able to hear their more confident peers using mathematical vocabulary. Research has shown that mixed-attainment groups benefit both high attainers, who become more secure in their maths knowledge through explaining their thinking to peers, and those less secure in their maths knowledge, through peer teaching.

Asking effective questions

The most skilled maths teachers can ask open questions to elicit students' current understanding. Skilful open questioning also allows students to articulate their current understanding carefully, and through this process either consolidate their understanding or come to realize where they have made a mistake. The following list offers a series of open questions that can be used whatever maths you are teaching.

- *How are these the same/different?*
- *What would happen if …?*
- *How else could you have done that?*
- *Why did you …?*
- *How did you …?*
- *How do you know that is correct?*

If you want students to check their solutions and consolidate their learning, ask them to explain how they reached their solution to a friend. Similarly, to support students in reflecting on their learning, you might ask the following:

- *What maths did you use to solve the problem?*
- *What new maths did you learn?*
- *What key words did you use?*
- *What was the most challenging part of the activity?*
- *What did you do when you got stuck?*
- *What other questions could you ask?*
- *Did this remind you of any other areas of maths?*

Differentiation

Differentiation is closely linked to inclusion: ensuring all students have access to the curriculum. This means that learning and teaching approaches must consider individual needs. Not all students will learn at the same pace or in the same ways.

This course supports the following approaches:

- **Differentiation by task** Content can be adjusted for some students to provide sufficient support or adequate challenge. The 'Fluency questions' in the Student Book are ramped, starting with questions aimed at less able students and finishing with 'stretch zone' questions. The latter are designed to extend more confident students and challenge them to think more deeply. 'Which method?' questions are also aimed at more confident students and require prior knowledge. 'Expert practice' questions are designed to be accessible to all, but self-differentiating depending on approach. For less able students, prioritize the questions in the Example-problem pair (EPP) grids.
- **Differentiation by outcome** This allows all students to tackle the same tasks, but with differentiated learning outcomes. There are three bands of differentiation for each learning objective: developing, secure, or extending. The differentiated outcomes are provided for each lesson in this Teacher's Guide. 'Secure' indicates that students have a secure grasp of the knowledge or skills specified. The band working towards 'secure' is 'developing', and the band moving past 'secure' is 'extending'.
- **Differentiation by support** This means providing more or less support as students are carrying out a task. Advice on this is provided for each lesson in this Teacher's Guide. For additional practice, support sheets are available on Kerboodle to give less able students further opportunities to reach a secure understanding of new or challenging concepts in their own time. These worksheets can be tackled independently or used in adult-led, small-group sessions.

Assessment

Assessment is an essential part of learning. Without being able to check progress, teachers and students will not be able to identify areas of strength and areas in need of development.

Each activity – group and individual – can be assessed through observation, questioning, and progress notes. Written or drawn responses for each activity can be assessed/marked using the school's marking policy; and chapter, end-of-term, and end-of-year judgements made about individual and class progress.

Feedback is a crucial aspect of assessment. This should be as positive and encouraging as possible, in which clear targets are identified. Involve students in assessment and target setting – assessment is done *with* learners, not done *to* learners.

Formative assessment

This takes place during learning and is used to address issues as they arise. This means learning and teaching can be modified during lessons to better meet students' needs. Feedback is ongoing.

Each activity within the Student Book provides opportunities for formative assessment and

	Learning outcomes		
Learning objective	**Developing**	**Secure**	**Extending**
Learners at this stage…	…are working towards secure knowledge and understanding but need more support to achieve this.	…have a secure knowledge and understanding.	…are working beyond expectations, and their knowledge and understanding can be stretched and challenged.
e.g. Understand and list multiples of a number	*List the multiples of a given number inside the times tables and use the term 'multiple' correctly*	*Understand multiples can be outside times tables up to 12*	*Understand that two multiples of a number add to give another multiple of the number*

feedback. You can do this by listening to explanations or paired discussions; observing students' workings; and assessing outcomes. Individual questions can be used to monitor understanding and identify misconceptions. These can be addressed as they are noted.

Summative assessment

This is used to measure or evaluate student progress at the end of a process – for example, when a chapter is completed or at the end of a year. Summative assessment compares students' attainment against a standard or benchmark.

The 'What have I learned?' pages at the end of each chapter can be used for summative assessment. You can record which questions each student is answering correctly and use this to measure individual attainment. It can also indicate how well the class is progressing through the work. In this way, the assessment can inform individual interventions (extra support for a student) or whole-class interventions (reviewing work that is not well understood).

How to support non-native English speakers

Ministries of Education at both local and national level are increasingly adopting the policy of English Medium Instruction (EMI), either for one or two subjects or across the whole curriculum.

In international schools, it is likely that students do not share a mother tongue with each other or perhaps the teacher. English is chosen as the medium of instruction to level the playing field and provide the opportunity to develop proficiency in an international language.

This does not mean that the maths teacher is expected to replace the English teacher, or to have the same skills or knowledge of English. However, they do need to become more language aware. This raises significant challenges, including:

- the teacher's knowledge of English
- students' level of English (which may vary considerably in international schools)
- resources that provide appropriate language support
- assessment tools that ensure that it is the content and not the language being tested
- differentiation that acknowledges different levels of proficiency in both language and content.

Language in the classroom

Using English in the classroom is very important as it provides exposure to an additional language (often a student's second or third), which plays a valuable role in language acquisition. The 'teacher talk' for purposes such as checking attendance and collecting homework does not have to be totally accurate or accessible to students. However, when teaching mathematical concepts, it is essential that the 'teacher talk' is comprehensible. The following strategies can help:

- simplify your language
- use short, simple sentences and project your voice
- paraphrase as necessary
- use visuals, the board, gestures, and body language to clarify meaning
- repeat as necessary
- plan before the lesson
- prepare clear instructions and check understanding.

Creating a language-rich environment

Providing a colourful and visually stimulating environment for students becomes even more important in the EMI classroom. Posters, lists of key words and structures, displays of students' work, and signs and notices in English all maximize students' exposure to English and, in big or small ways, contribute to their language acquisition.

Planning

In your planning, identify each language demand (LD). You will need to think about what language students will need to understand or produce, and decide how best to scaffold the learning to ensure that language does not become an obstacle to understanding the concept. This kind of language support (LS) goes beyond the familiar strategy of identifying key vocabulary.

Support for listening and reading

Listening and reading are receptive skills, requiring understanding rather than production of language.

If students need to listen to or read in English, ask yourself the following questions:

1. Do I need to teach any vocabulary before they listen/read?
2. How can I prepare them for the content of the text so that they are not listening 'cold'?

3 Can I provide visual support to help them understand the key content?
4 How many times should I ask them to listen/read?
5 What simple question can I set before they listen/read for the first time to focus their attention?
6 How can I check more detailed understanding of the text? Can I use a graphic organizer (e.g. tables, charts or diagrams) or gap-fill task?
7 Do I need to differentiate the task for developing and extending students?
8 Can I make the tasks interactive through groupwork or games?
9 How can I check their answers and give feedback?

Support for speaking and writing
Speaking and writing are productive skills and may need more language input from the teacher. You will need to think in detail about what language the task requires (language demands, LD) and what strategies you will use to help students use English to perform the task (language support, LS).

Ask yourself the following questions:

1 What vocabulary does the task require? (LD)
2 Do I need to teach this first? How? (LS)
3 What phrases/sentences will they need? Think about the language for learning maths (e.g. predicting and comparing). What structures do they need for these language functions? (LD)
4 While I am monitoring this task, is there any way I can provide further support to less confident students? (LS)
5 What language will students need to use at the feedback stage (e.g. when they present their task)? Do I need to scaffold this? (LD, LS)

Teaching vocabulary and structures

Vocabulary
Learning key vocabulary is central to EMI, and 'learning' means more than simply understanding the meaning. Knowing a word also involves being able to pronounce it accurately and use it appropriately. Aim to adopt the following strategies:

- Avoid writing a vocabulary list on the board at the start of a topic and 'explaining' it. The vocabulary should be introduced as and when it arises. This helps students associate the word or phrase with the concept and context.
- Record the vocabulary clearly on the board. Check your pronunciation and spelling.
- Give students a chance to say the word once they have understood it. The most efficient way to do this is through repetition drilling.
- Use visuals whenever possible to reinforce students' understanding of the word or mathematical concept.
- Advise students to record the vocabulary systematically in their glossaries under chapter or topic headings.
- Remember to recycle and revise the vocabulary.

Structures
Students will need to use phrases and sentence frames to discuss or write about their learning in maths, including these structures:

X is the same as Y.

The next number in the sequence is ... because ...

I predict that X will happen.

If X happens, then Y happens.

The next step is …

Build up these banks of common maths phrases and remind students to record them. You do not have to focus on grammar here as the language can be taught as 'chunks' rather than specific grammatical structures.

Component overview

Student Books

The Student Books are textbooks for students to read and use. They include everything you need to deliver the course to your students, guide their activities, and assess their progress.

Student Book	Typical student age range
Student Book 7	Age 11–12
Student Book 8	Age 12–13
Student Book 9	Age 13–14

Teacher's Guides

There are three Teacher's Guides, corresponding to the three Student Books. Each Teacher's Guide includes:

- advice on delivering maths lessons effectively for EAL students

- a brief introduction to each chapter, including pre-requisite knowledge and mathematical concepts that will be revisited, an introductory activity using a picture prompt, teaching strategies, common learning misconceptions and real-world applications of maths
- guidance on teaching each Student Book topic, including student learning objectives and outcomes, recommended scaffolding, answer keys and approaches to problem-solving.

Digital

Kerboodle online learning (www.kerboodle.com) provides engaging digital books, lesson resources, and a comprehensive assessment package.

Digital books

- **For the teacher:** You can access the Student Books and Teacher's Guides as digital books. The digital books show the course content on screen, making it easier for you to deliver engaging lessons. A set of tools (e.g. sticky notes, bookmarks, pen features, zoom in, and spotlight text) is available to personalize your digital book and make notes. You can share your notes or hide them from view.
- **For the students:** Students can access the Student Books as digital books for use at home.

Resources

- Videos – on each topic, also integrated into students' adaptive learning journey
- Exercise handouts – useful visual aids and additional scaffolding for answering the Student Book questions
- Support worksheets and answer keys – extra fluency questions for students who need more practice at developing-level questions
- Example-problem pairs worksheets – additional practice and support for completing EPP grids
- Vocabulary quizzes – for each chapter, to assess students' understanding of key terms
- Mapping to the English National Curriculum, Cambridge international curriculum, and Oxford International Curriculum
- Guidance on how the series supports progression to further study at iGCSE
- Letters to parents/carers to introduce the course and offer guidance on home learning.

Assessment and adaptive learning journey

With a Kerboodle login, you can access all the quizzes and tests. First, you will need to import class registers and create user accounts for your students. Once your classes are set up, you can assign students assessments to complete.

Our assessment model combines formative and summative practices. An additional element is regular, low-stakes quizzing aimed at helping students retain new concepts. The formative assessment comprises:

- My self-study quizzes at the end of each topic ask students questions that are relevant to the learning objectives they have just covered. Students' scores will generate either a 'developing to secure' next-step intervention, or a 'secure to extending' next-step intervention. The teacher will also see a breakdown of how students are performing against each of the learning objectives.
- Formative tests which cover content from the whole chapter. Similarly, students will be assigned a next-step intervention according to their score.

Quizzes and tests are auto-marked. Following either assessment type, students are offered personalized next steps. They can consolidate their knowledge if they are at a developing level, or challenge themselves if they have demonstrated secure knowledge.

At the end of each chapter, there is a paper-based summative assessment designed to evaluate understanding of the whole chapter.

Reporting and insights

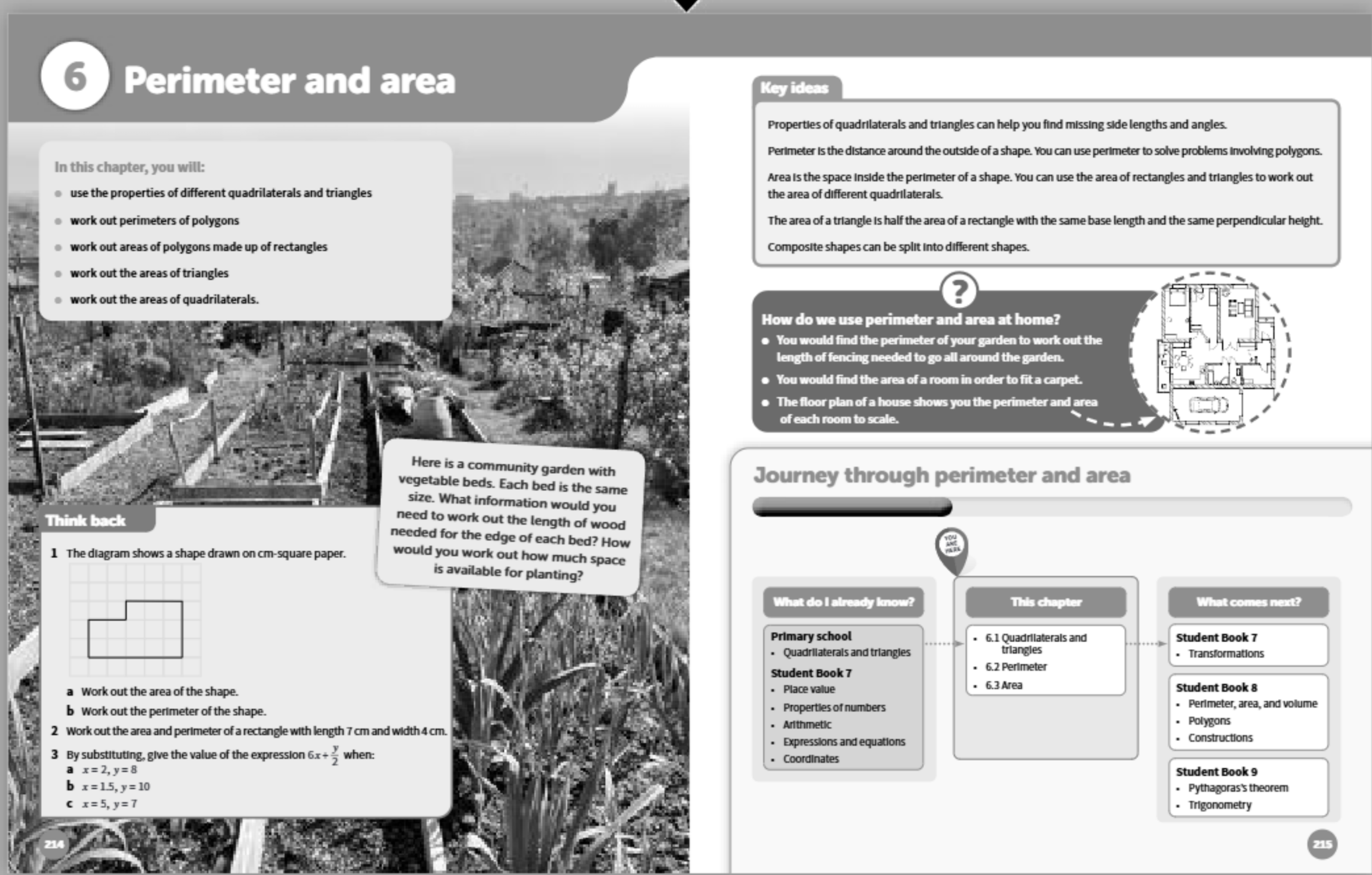
The formative assessment data will feed reporting on Kerboodle and give insights into strengths and areas for development. The data is broken down into learning objectives, and will support you in diagnosing learner needs and focusing your intervention accordingly.

Tour of a Student Book

Chapter opener

This explains to students what is coming up in the chapter. The ‘Learning journey’ map shows clearly what maths students should already know from previous learning, the new topics they will study in the chapter, as well as the next steps in their maths learning.

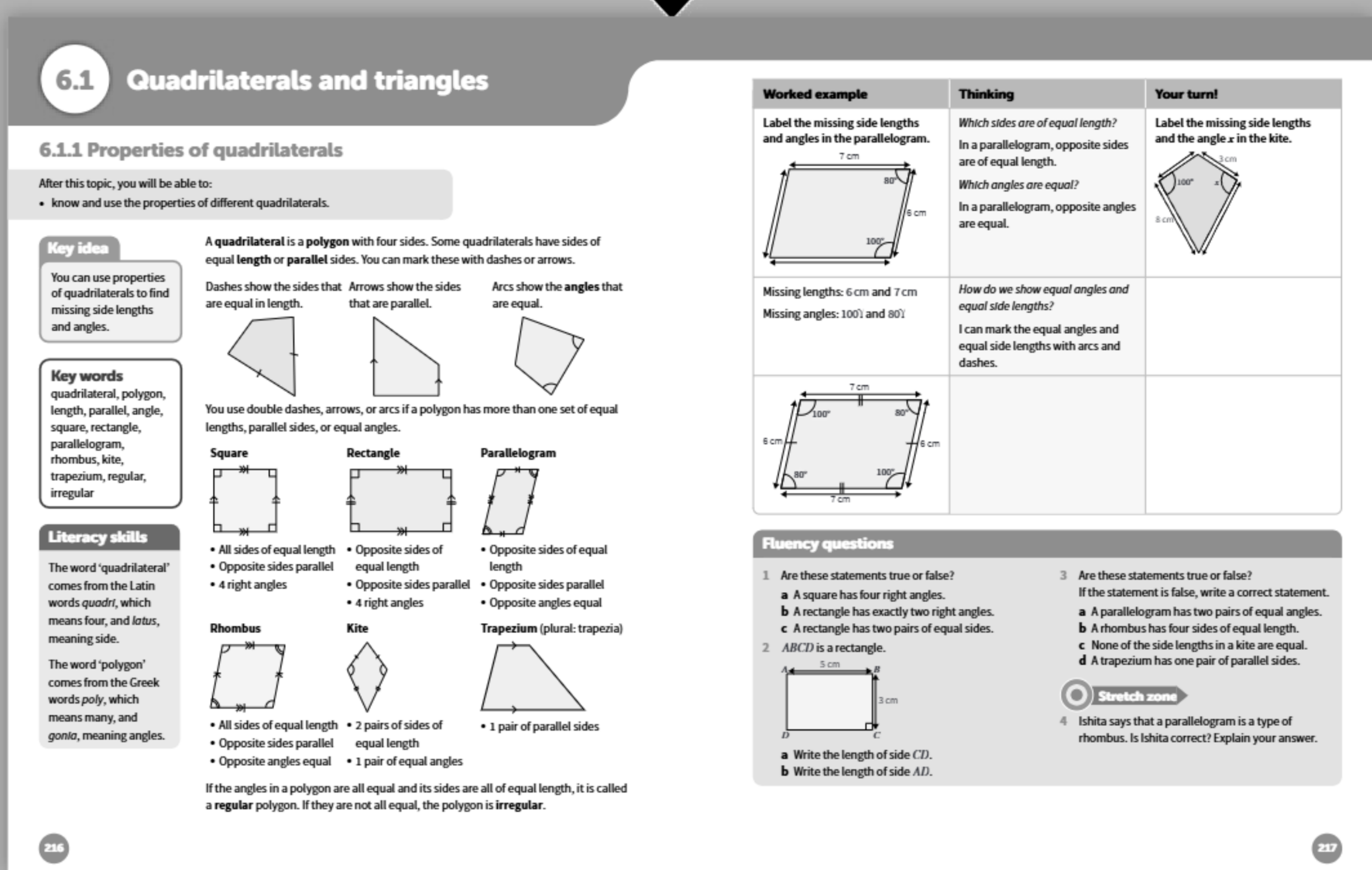
The ‘Think back’ questions help students recall existing knowledge. This feature will warm up their thinking and alert you to any gaps in their learning before carrying on.



Lesson pages

These pages guide students through a particular topic in each chapter. Simplified language is clear and accessible for English language learners, to ensure that a developing understanding of English does not get in the way of grasping key concepts. Skills boxes and fluency questions can then be used to check students’ understanding of what they have just read and to stretch their thinking further.

Learning objectives for the lesson are clearly set out at the start and summarized in the Key idea box.



Key words boxes highlight the main maths vocabulary for the lesson. These words are also found in the Student Book glossary.

Intelligent practice, Which method?, Expert practice

After the lesson pages for each topic, there are three different types of exercise for students to apply and practise the maths they have just learned:

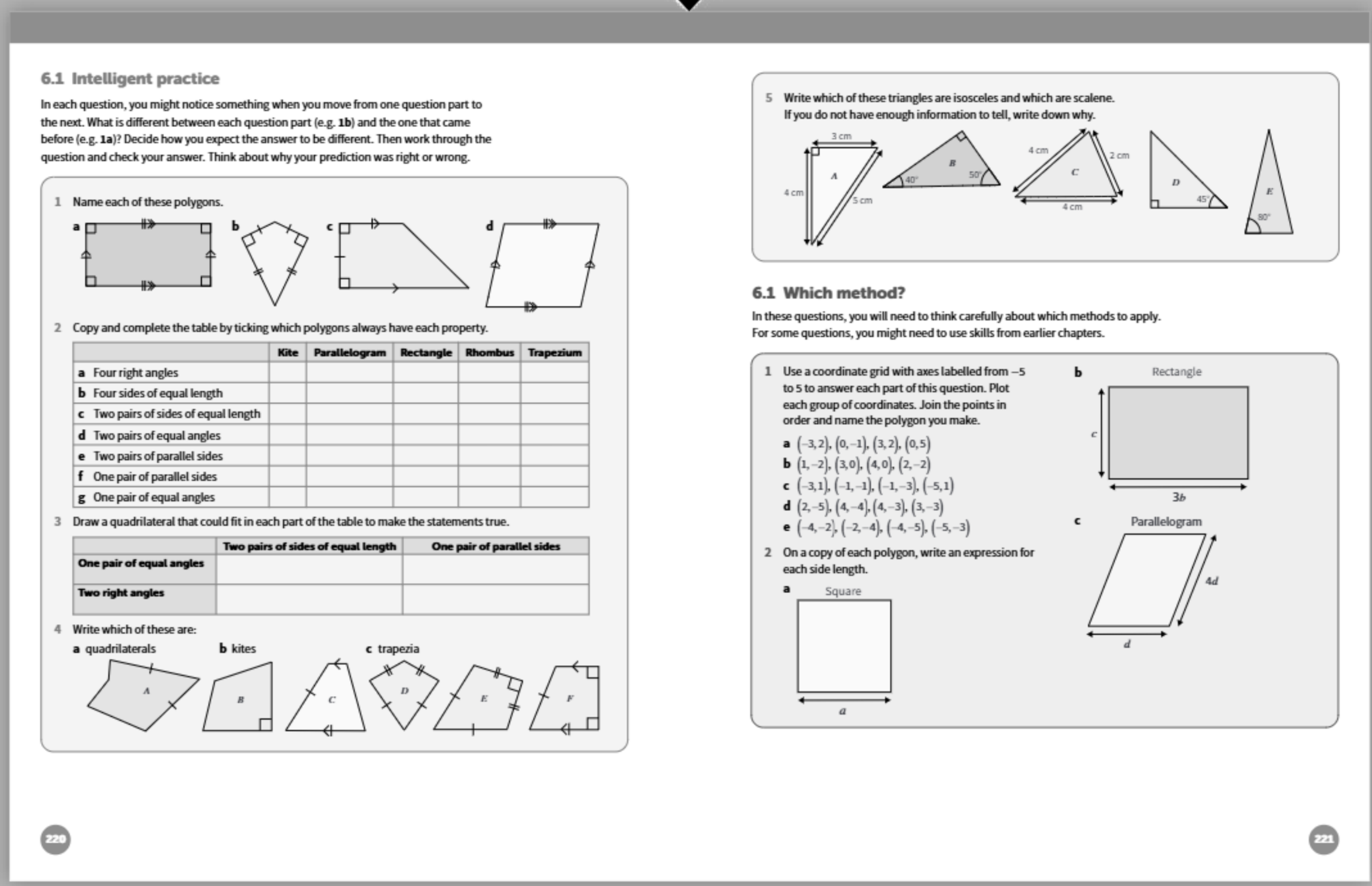
1 Intelligent practice

2 Which method?

3 Expert practice

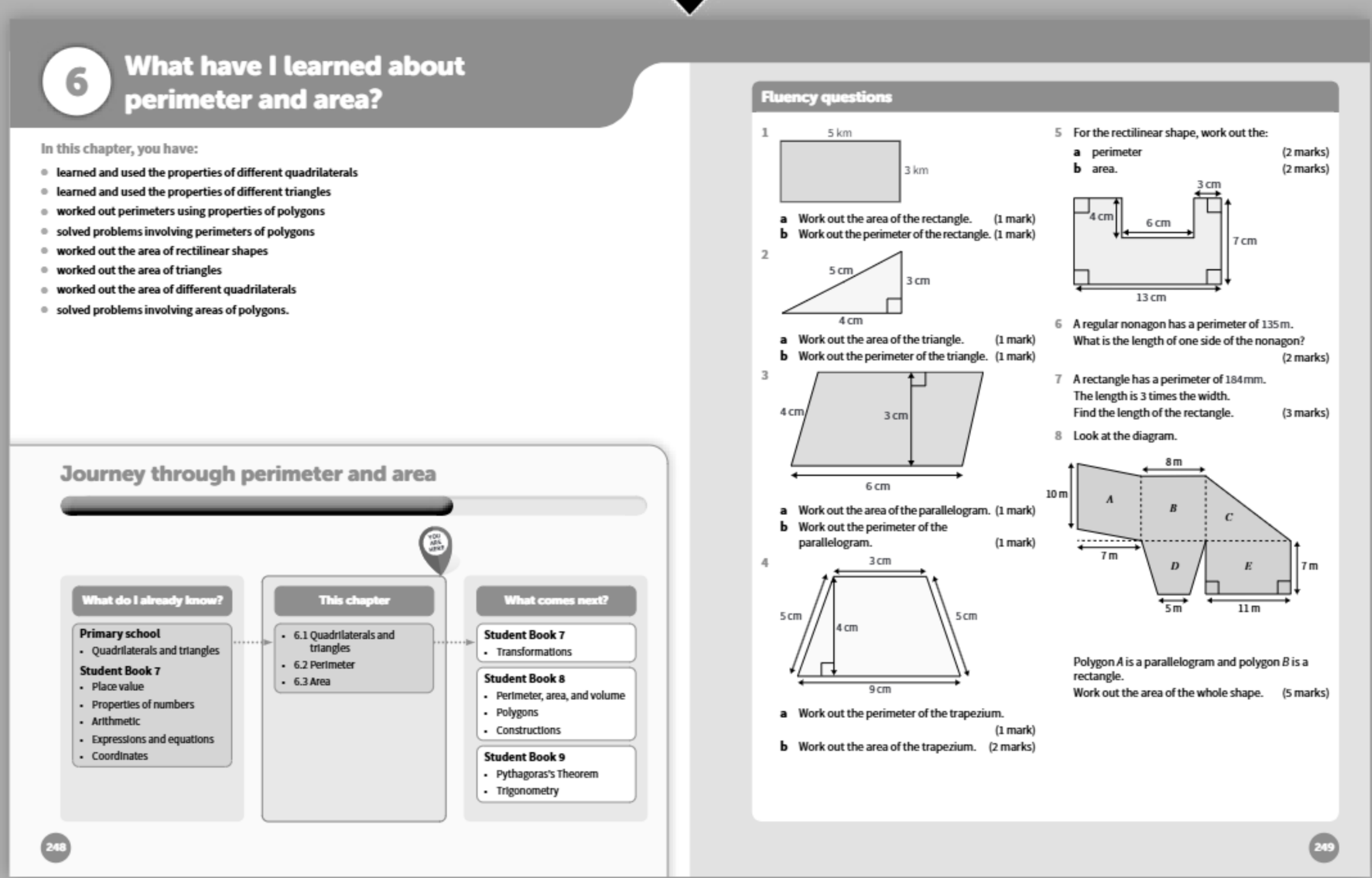
Each exercise works in a particular way to help the brain make connections, remember the topic, and recognize when to use it.

Find more information about these exercises on p.xiii of this Teacher's Guide.



What have I learned? pages

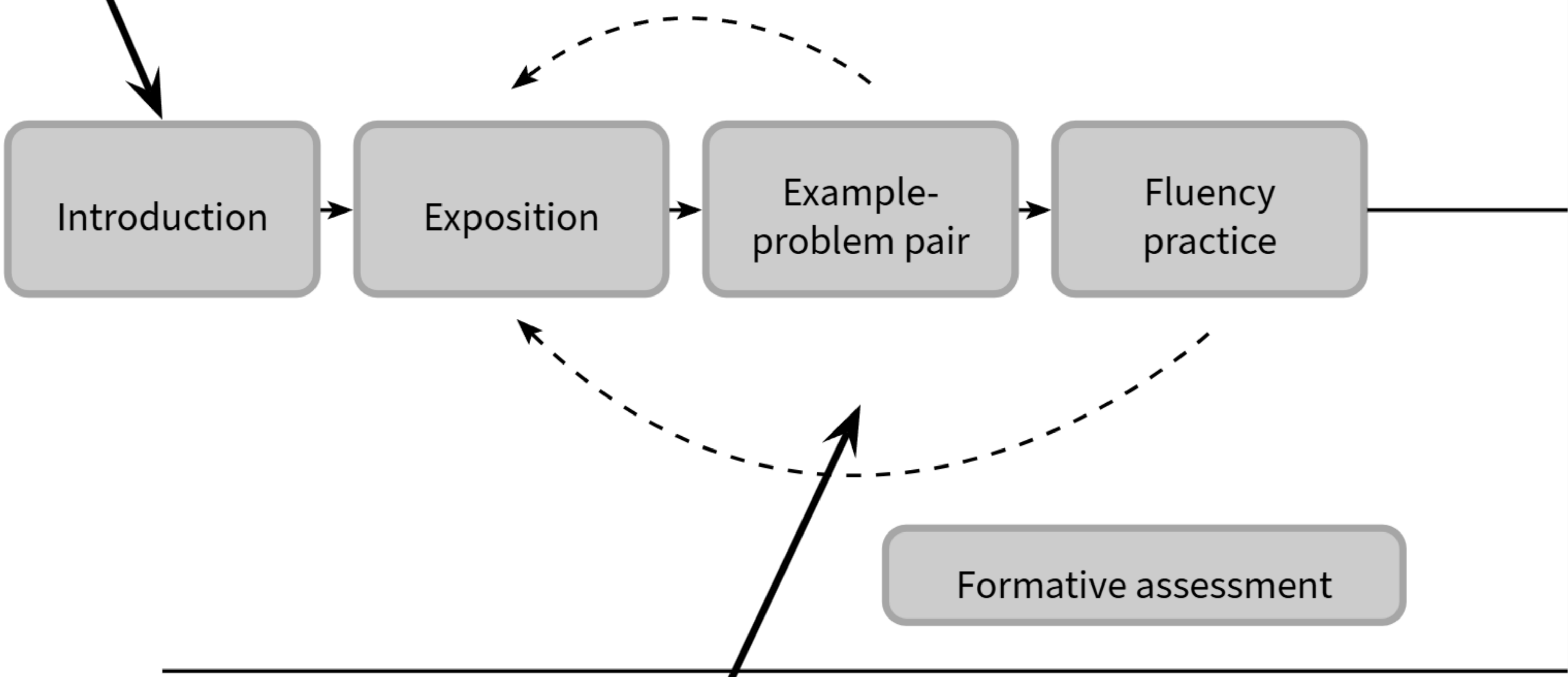
These pages summarize the content that students have learned so far and show how they have progressed in their learning journey. Each chapter concludes with exam-style questions to test how well students have learned and understood the topics, and to keep track of their overall progress.



The Learning Episode

Introduction
To stimulate interest in a way that helps students feel confident that they can be successful in the maths that will follow.

- **Link to the big picture**
  The big picture in the chapter opener aims to provide both a purpose to students' study and a platform upon which to build strong memories (e.g. visual examples).
- **Tell a story**
  This will make the information easier to comprehend and remember (e.g. personal reflection – memories of learning it as a child or how you used the knowledge recently).
- **Provide a question hook**
  This should be short and snappy, to spark student interest. You can use the questions linked to the big picture or real-world application in the chapter opener.
- **Discuss the etymology of key words**
  This will make it easier to remember the words' meaning, showing students how words connect to other areas of maths and the world. You can use the Literacy skills boxes throughout the Student Book.



Example-problem pair
To model how to do a new method in a way that allows students to be actively involved and attend critical elements.

1. **Silent teacher**
   Run through the example without verbalizing any questions, but pausing and turning to students to cue them to think about what has happened.
2. **Narration**
   Go through the example again, this time using thinking prompts to draw students' attention to critical elements of the working out.
3. **Read the maths**
   Students use the thinking prompts to add annotations to the example (downloadable worksheets are available on Kerboodle) so that it makes sense.
4. **Your turn**
   Students complete the question using the model example to support them with answering the question.
5. **Share learning**
   Use a visualizer and examples of student work to highlight examples of good practice, misconceptions, etc.

Intelligent practice

These are sequences of questions that enable students to gain practice in carrying out a mathematical method, while providing opportunities to think mathematically.

1 Model the relationship
Model the *Reflect, Expect, Check, Explain* process first using an example question.

2 Silent practice
Students complete questions in silence, allowing them to think before they ask for help and make connections at the points at which they are ready.

3 Paired discussion
Students rehearse and modify their explanations, and listen and learn from others.

4 Discuss relationships
Reveal the answers and delve deeper into one or two relationships.

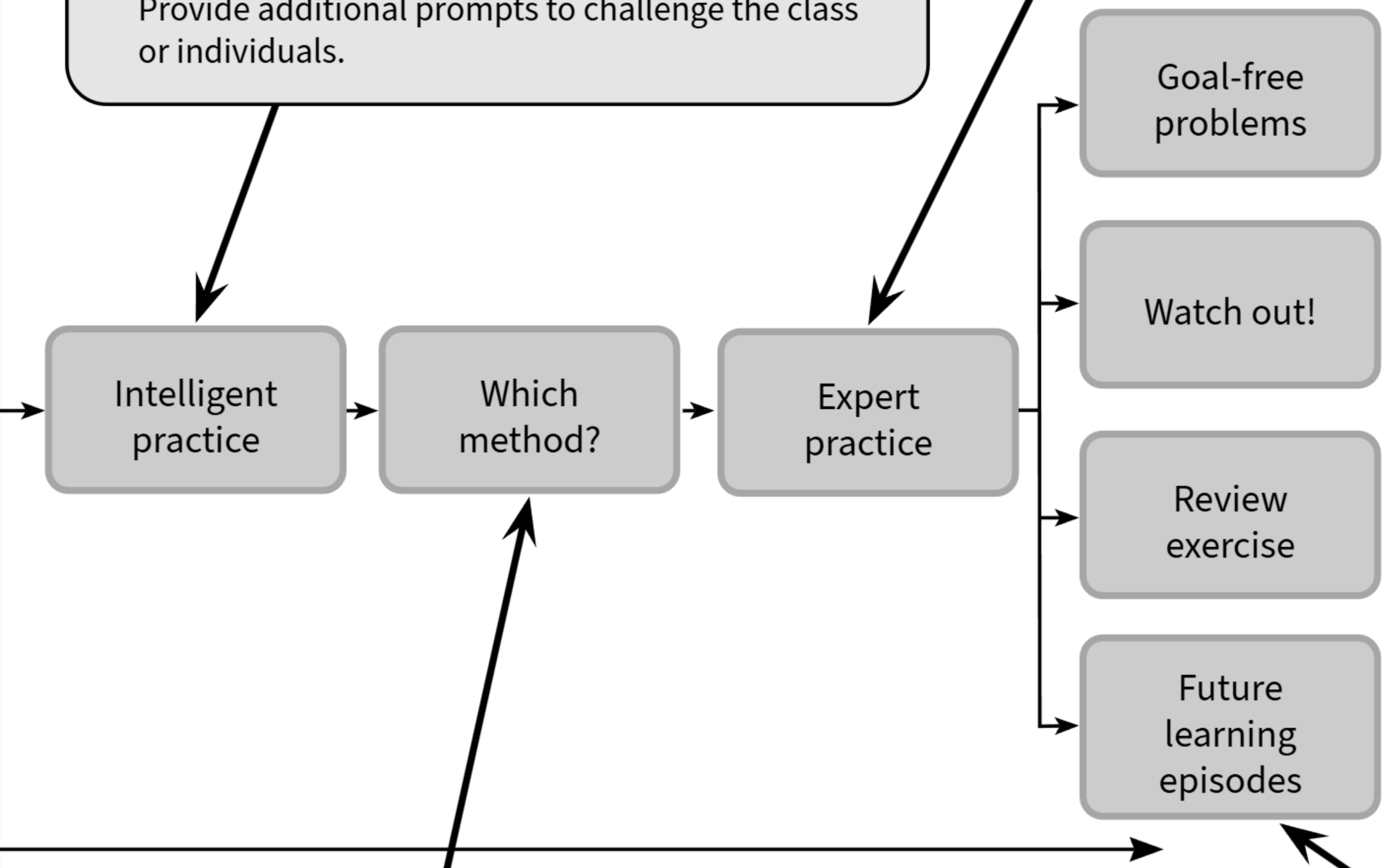
5 Prompt to delve deeper
Provide additional prompts to challenge the class or individuals.

Expert practice

These questions are less structured than in the Intelligent practice exercise. They provide practice of a key method, while providing opportunities for students to think more deeply and with greater purpose.

Five principles:

1. Students need to experience early success to support motivation.
2. There must be plenty of opportunities to practise the key procedure.
3. The practice should feel different to prevent boredom and allow students to see things from a different angle.
4. Opportunities must exist for students to make connections, solve problems, and think more deeply.
5. The focus is always on the practice, allowing students to make connections at their own pace.



Goal-free problems
Working on these types of problem is more likely to secure the understanding of concepts and connections due to reduced pressure on working memory. Goal-free problem exercises are provided at the end of each chapter in this Teacher's Guide.

Watch out!
This exercise introduces students to common misconceptions in the topic areas they have just been studying, at a point when their understanding is secure enough to understand them. This activity also supports students' reasoning skills. They are provided as downloadable worksheets on Kerboodle.

Review exercise
These end-of-chapter fluency questions, with mark scheme, encourage students to reflect on their learning and test themselves to see if they have understood the content.

Which method?

This exercise ensures that students know when and when not to apply a given method. It also provides retrieval opportunities for previous learning. These questions:

- include ideas from the current Learning Episode along with ideas from previously covered Learning Episodes, which on the surface appear similar
- support students in identifying the strategy needed to answer a question
- highlight the importance of reading the question
- allow students to appreciate exactly what it is about a question that requires a given method.

1. Model how to approach a question and work out what method is needed.
2. The *Reflect, Expect, Check, Explain* process could be used to support students' thinking here. They can *reflect* on what is familiar about the question and what links they can see to other questions or topics, make an *expectation* about what method is needed, try the question and *check* their answer, and then *reflect* on whether that was the correct method and how they might approach similar questions in future.

Reflect, Expect, Check, Explain (RECE)

The Reflect, Expect, Check, Explain (RECE) model is a pedagogy that encourages student self-reflection when answering questions. RECE exploits the Self-Explanation Effect, which shows how inviting students to narrate their thought processes while completing tasks can improve understanding and long-term recall. In maths, it is a powerful way to form connections between concepts and help students reflect on how what they already know can help them understand what they do not.

Following the Reflect, Expect, Check, Explain prompts is particularly impactful when students are practising questions, where only one or two elements change between questions. RECE supports students to **reflect** on what they know and on the basis of this make predictions about what they would **expect** to happen. After completing the question, students **check** their answer and then attempt to **explain** why the answer is, or is not, what they expected.

The RECE model can be used in a variety of ways across a Learning Episode and this Teacher's Guide will prompt you where it is especially useful – for example, during direct modelling as part of an EPP-fluency cycle or Intelligent practice. Using RECE promotes more general reflective thinking, which has benefits throughout students' learning journey and ultimately supports them to become more confident mathematicians.

The RECE model has been designed to be simple to use, but like any new approach it is important to model first to students. This ensures that they understand how it works and can try it out themselves with support. Here are some prompts that you can use with students when introducing RECE:

- *First, read the question. What do you notice? Based on previous questions that are similar, what has changed and what is the same?*
- *What would you expect to be the answer to this question? What do you need to do next that is the same and different? Will the answer increase/decrease? etc.*
- *If the answer surprised you, can you explain why? If it did not surprise you (you answered correctly), how could you explain what you did to someone who does not understand yet? If you did not notice the link between the questions or explain the relationship before, can you do so now?*

Addressing misconceptions

The Education Endowment Foundation (EEF) defines a misconception as 'an understanding that leads to a systematic pattern of errors'. That is, misconceptions can arise when a generalization has been applied outside of the context in which it is useful.

It is important that teachers have knowledge of possible misconceptions so that they can plan to prevent them before they arise, as well as uncovering and addressing them when they do.

At the start of each chapter in this Teacher's Guide, there is information about the common difficulties and misconceptions that may arise during teaching, with suggestions of how to avoid them. These are also discussed in context during the exposition of each Learning Episode.

Some things to consider when planning to address misconceptions:

- Early in the Learning Episode, focus only on the misconceptions that you **know** students hold.
- Consider how a misconception might have developed. What generalizations has the student misapplied and what counter examples could you use to challenge their belief?
- Plan opportunities for discussion, either as a class or in small groups.
- Use examples, questions, models, and images that allow students to make accurate generalizations and conclusions.
- Compare examples and non-examples of a concept, as well as standard and non-standard representations.

1 Similarity and congruence

Introduction to chapter

This chapter builds on the previous geometry work from Student Books 7 and 8 and links the angle properties of shapes and transformations. Students formalize their mathematical notation and language around shape before looking at similarity. The section on similarity links back to multiplicative relationships from Student Book 7, Chapter 8, and enlargements from transformations in Student Book 7, Chapter 9. This is followed by congruence, which links to polygons from Student Book 8, Chapter 8, and congruent transformations from Student Book 7, Chapter 9. This chapter prepares students for more complex geometry in later chapters (most notably, the general triangle and labelling hypotenuses).

Core concepts

- Proportionality
- Shapes and solids
- Orientation and location
- Transformations

What have students already learned?

- Comparing and classifying geometric shapes based on their properties
- Finding unknown angles in any triangles and quadrilaterals
- Finding different pairs of numbers that satisfy the same multiplicative relationship
- Recognizing a translation, reflection, rotation, or enlargement and knowing which features of an object are affected

What will students revisit in this chapter?

- Solving problems involving similar shapes where the scale factor is known or can be found
- Knowing and using the properties of different triangles

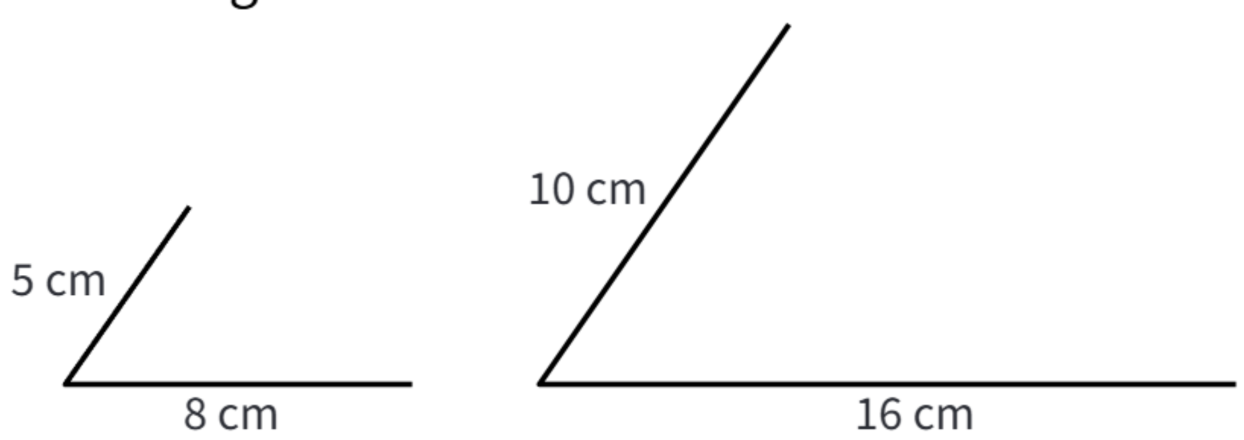
Getting started p.2

Direct students' attention to the inset photos in the Student Book and encourage a discussion:

- *For each of A, B, and C, describe in words how the original image of the statue (O) has been resized.* (A: The image has been made wider but not taller. B: The image has been made taller but not wider. C: The image has been made taller and wider.)
- *Image A is 1.5 times wider than the original image but is the same height as the original image. Image B is 1.5 times taller than the original image but is the same width as the original image. What can you say about image C?* (Image C is 1.5 times taller *and* wider than the original image.)
- *Which of the three resized images looks like the most accurate representation of the original image, O?* (Image C)
- *What scale factor is the image O enlarged by to obtain image C?* (1.5)
- *A fourth resized image is created. This new image is twice as wide as the original and twice as tall as the original image. Is this image an enlargement of image O? If so, by what scale factor?* (Yes, this is an enlargement of image O by scale factor 2.)

Teaching strategy

Support students' understanding by carefully explaining that angles stay the same under enlargement. Demonstrate this practically by drawing an angle on the board like this. Show that if you double the length of the lines, the angle does not change.

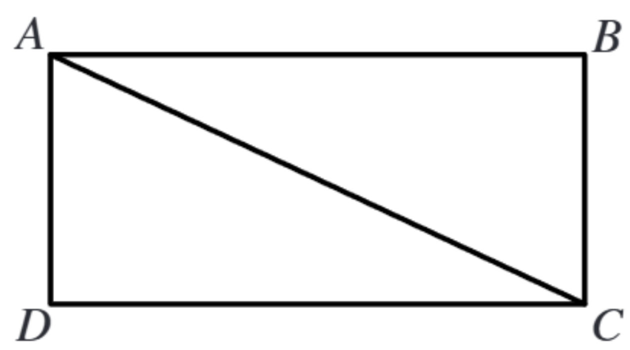


This relates to students' knowledge about squares (they have four right angles whatever the length of the sides) but students are not used to thinking about this in terms of similarity.

Even if students understand why certain conditions are sufficient for congruence, recognizing when this is *not* the case is much more difficult.

Common learning misconceptions

In this chapter, many of the common learning misconceptions align with the common learning misconceptions in previous chapters on geometry. Students sometimes have difficulties using letters to denote vertices when they are used to representing variables with letters. They might also find it difficult to 'see' one shape on top of another when shapes are layered rather than positioned next to each other. This is a barrier when it comes to some of the similarity work later in this chapter. For example, this image shows three shapes: the rectangle $ABCD$, the triangle ABC, and the triangle ACD.



Broader context

The formal notation in this chapter might be students' first experience of technical written communication that is different from standard sentences. This ability to read a more succinct style of written communication is applicable to music notation, shorthand, and formal logic. The ideas of similarity also link to scale drawings and have applications in computer-aided design and schematics. Testing for congruence develops students' logical reasoning skills and ability to reflect on incomplete information.

1.1 Notation and naming

Students practise labelling points with capital letters, labelling line segments using the endpoint labels (i.e. line segment AB is the straight line that links point A and point B), and labelling angles using three capital letters. Students will then formalize notation for equal sides and angles in polygons by using dashes and double arcs. They will also revisit using a small square to denote an angle of 90°.

Students will also consider the differences between sketching shapes and constructing them (linking to Student Book 8, Chapter 9).

	Learning outcomes		
Learning objectives	**Developing**	**Secure**	**Extending**
Label sides and angles of triangles and other polygons	Use capital letters to label vertices and pairs of letters to label line segments with those end vertices Use triple letters to label angles so that angle ABC means the angle at B, between line segments AB and BC Know that angle ABC is the same as angle CBA *e.g. Measure angle CAB.* Determine which side of a triangle is the longest	Know and use the notation $\angle ABC$ or $A\hat{B}C$ to represent angle ABC Use a string of letters to denote a polygon where vertices are written in the order in which they are joined, so polygon $ABCDE$ is not the same as $ACBDE$ *e.g. Shape ABCD is translated. Write the name of the translated shape. Write the vertices in the same order as the matching vertices in ABCD.*	Use the notation of the general triangle with sides a, b, and c opposite angles A, B, and C, respectively *e.g. Write an expression for the area of the right-angled triangle ABC.*

Understand and use terms and notation for right-angled triangles and triangles with equal angles	Recall that an isosceles triangle has a pair of equal angles and an equilateral triangle has all equal angles Recognize that right angles are denoted with small square symbols Recognize that equal lines or equal angles are denoted with pairs of dashes *e.g. Which of the triangles are:* *a) right-angled* *b) isosceles* *c) equilateral?* Know and use the term hypotenuse for the side opposite the right angle in a right-angled triangle and label the hypotenuse of a right-angled triangle where the height is perpendicular to the width of the page *e.g. Write the name of the side that is the hypotenuse of each of these right-angled triangles.*	Identify a right angle in a triangle when given the hypotenuse Identify the hypotenuse of a right-angled triangle in any orientation	Use correct terms and mathematical notation to explain why a right angle in a triangle must be the largest angle and the other angles must be acute Identify contextually whether or not a line segment that is common to two triangles is a hypotenuse *e.g. Which triangle has BD as a hypotenuse?*

Tier 2 vocabulary	Tier 3 vocabulary
acute angle, area, equilateral triangle, hexagon, isosceles triangle, line segment, mathematical, obtuse angle, point, polygon, quadrilateral, reflex angle, right-angled triangle, sketch, triangle	hypotenuse, vertex (plural: vertices)

Classroom resources

Equipment
You will need:

- pencil and ruler

Digital resources

My self-study quiz, Example-problem pairs, Exercise handout, Extra fluency questions

1.1.1 Labelling polygons p.4

Objective

Students will learn how to:

- label sides and angles of triangles and other polygons.

Ask students: *How do we describe shapes?* Students might mistakenly say 'line' when they are talking about a 'side' of a shape. Explain that mathematical definitions are more precise than the language we use in regular conversations. Emphasize that using mathematical language correctly is important. Introduce the idea of a point as a position within space. Mark a point, P, on the board. There is a subtle difference between an actual point and the mark we make on a page to demonstrate a point. You could discuss this in more detail later when looking at lines.

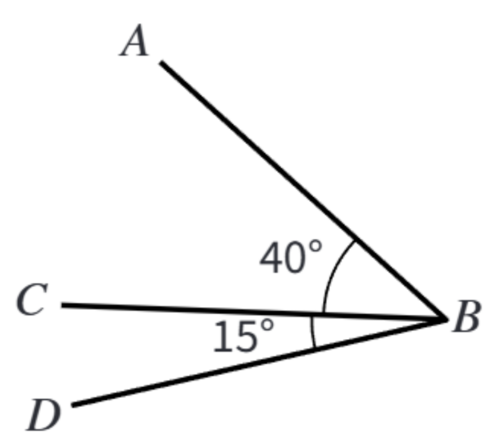
Build on point P by adding a point Q and joining them with a line segment. Discuss how the line segment PQ is the straight line that connects points P and Q. Emphasize to students that line segments are always straight lines, not arcs or curves. This precise language does not feature in our day-to-day speech when we discuss shapes, but it allows mathematicians to be very clear. In a Euclidean sense, a line extends infinitely in both directions, whereas a line segment has finite length. This distinction might not seem important to students at this stage, but the concept will become important in their future studies.

Extend the diagram further by adding a point R to make a triangle as in the Student Book. Explain that in the same way as we describe line segments using points, we describe triangles, so triangle PQR is the triangle with vertices at points P, Q, and R. You could prompt students to consider triangle QPR, triangle RPQ, and so on. Emphasize that these are all the same triangle because they contain the same points. However, by convention, we write the notation for polygons alphabetically where possible.

On this same diagram, identify that angle PQR still uses the three points but refers to an angle. Students learned in previous chapters that angles are a measure of a turn. In this notation, angle PQR is the measure of the turn between line segments PQ and QR. Showing this turn using a visualizer (e.g. by placing a pencil on the line segment PQ and rotating it about point Q until the pencil lies on line segment QR) or using dynamic geometry software will be more effective than a description.

Explain that 'angle PQR' could refer to either the acute angle or the reflex angle. We use an arc to show which angle we are referring to. In polygons, if neither angle is marked with an arc, we conventionally assume that the notation refers to the interior angle.

Show students the diagram below to demonstrate why we use three points to describe an angle.



In this diagram, there are three angles displayed. Angle ABC is 40°. Angle ABD is 55° because this is the turn from AB to BD rather than the turn from AB to BC. You could ask students to name the angle with a value of 15°. You could ask students working towards extending outcomes to add a point E to the diagram, without using a protractor, so that angle DBE is 15°. E can be anywhere along the line segment BC to make this true.

This subsection introduces the two main ways that we write angles: with a ∠ symbol before the three points, or with a hat over the middle letter, for example $R\hat{Q}P$. These symbols make our written communication more succinct and efficient. Explain to students that these symbols will be used interchangeably during this chapter.

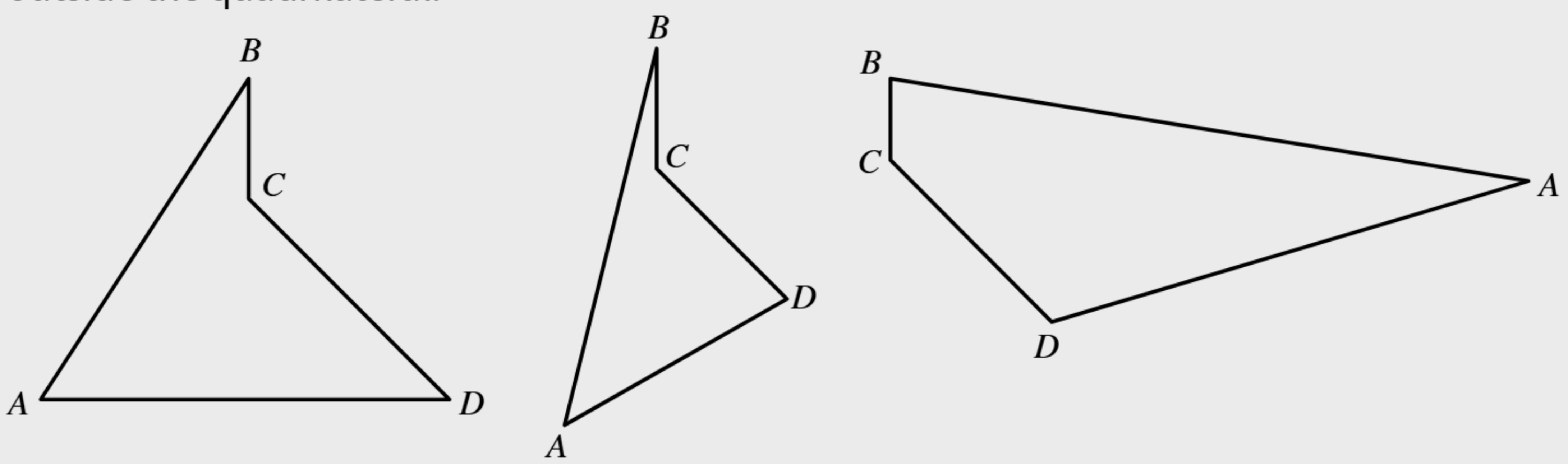
You could show students examples of triangles labelled with points and given angles and ask them to write down the name of an angle with a particular measurement. Students could do this by marking the angle and then the relevant sides to put together the order of the three letters required. Some students may not need to break such questions into steps, but some will. It is quite common for students to have difficulties with questions involving interpretation and identification as opposed to calculation.

Extend the naming convention to all polygons. This might seem simple, but students can find it difficult to move between an image of a shape and a description.

Example-problem pair

This question focuses on translating properties of shapes written in formal notation into diagrams. In the worked example, this is explained by breaking down the question into just one property at a time. By starting with just the angle, and then thinking about side lengths, we can build up to a completed diagram.

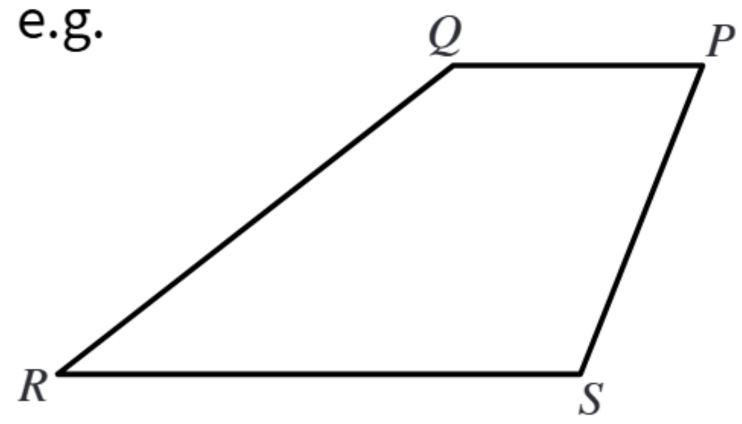
Discuss the different ways in which we can finish diagrams.
For example, in these three diagrams, only the first two are valid answers because, in the third diagram, the reflex angle BCD occurs outside the quadrilateral.



Ask students to try 'Your turn!'.

Your turn! answer

e.g.



Introduce the general triangle. Emphasize the difference between capital letter notation for vertices and lower-case letters for sides.

After students complete the Fluency questions, use these to check students' understanding before moving on.

1.1.1 Fluency questions: answers p.5

1 AB or BA

2 **a** ABC, BCA, CAB, CBA, ACB, or BAC
b B
c AB or BA
d BAC or CAB

3 **a**, **c**, and **d**

4 $\frac{1}{2} \times BC \times AB$ (or equivalent)

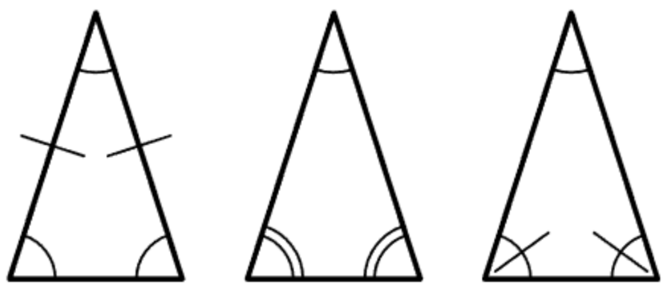
1.1.2 Right-angled triangles p.6

Objective

Students will:

- understand and use terms and notation for right-angled triangles and triangles with equal angles.

This subsection begins with dash and arc notation to show equality. Explain that double arcs and dashed angles mean that two angles are the same size and, as with our written angle notation with the hat accent and angle symbol, these two are interchangeable. Revisit isosceles triangles and how we know if a triangle is isosceles. For example, all three of these diagrams give us enough information to state that the triangles are isosceles.



So far, students have looked at the relationships between sides and the relationships between angles separately. Trigonometry will be the first time the proportions between angles and sides are explicitly defined and used for calculations. However, here we define the hypotenuse as the longest side in a right-angled triangle. This is a small but vital detail at the beginning of a more complex thread that will be developed later on.

Explain and how the hypotenuse will always be the side opposite the right angle in any right-angled triangle. This links to the general triangle in the previous subsection, where opposite sides used the lower-case letter of the angle. You could show students a selection of triangles, with only one right-angled triangle. Ask them to explain why only one of the triangles has a hypotenuse (because only one of the triangles has a right angle).

Example-problem pair

This question requires students to identify equality in the properties of the given triangle. Students need to remember that there are different ways to show equal angles. The fact that the triangle has a right angle is not relevant to part **a**. Reinforce working systematically. Explain that even where there are not multiple parts to a question, mathematicians will always work on individual aspects of a solution one at a time before bringing it all together. Ask students to try ‘Your turn!’.

Your turn! answers

a Sides XZ and YZ are equal. $\angle ZXY$ and $\angle ZYX$ are equal.

b No, this triangle does not have a hypotenuse.

After students complete the Fluency questions, use these to check students’ understanding before moving on.

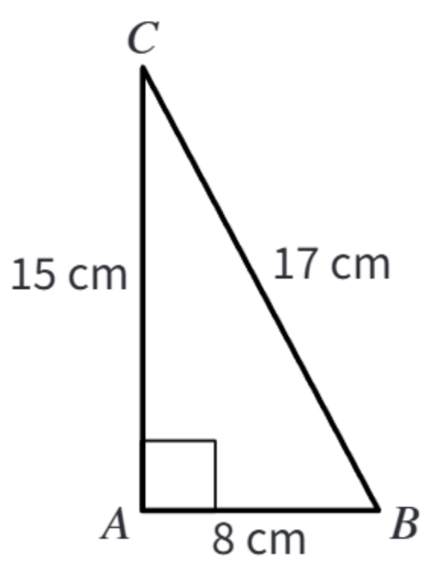
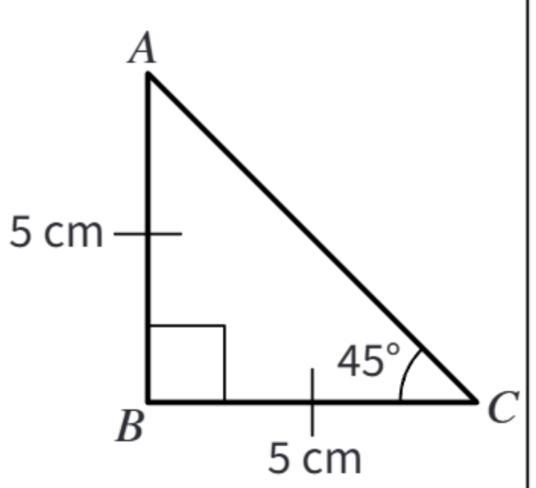
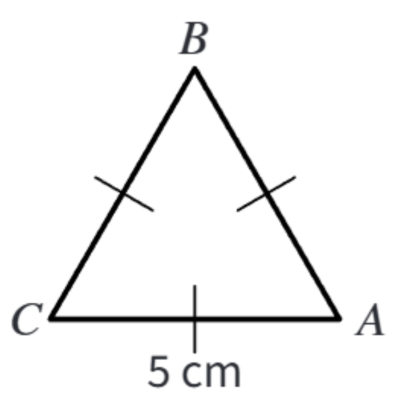
1.1.2 Fluency questions: answers p.7

1. It is an isosceles triangle. It has two equal sides, and/or two equal angles.
2. Both are correct. An equilateral triangle has three equal sides and three equal angles.
3. Angle *PRQ*. It is opposite the hypotenuse.
4. *RT*. It is opposite the right angle.
5. **a** **i** largest **ii** acute
   b The right angle uses up half of the 180° sum of all the angles, so this angle must be the largest angle. The other two angles share the remaining 90°, so they must both be acute.
6. Triangle *PQS*. *PQ* is opposite the right angle in that triangle, but *PQ* is not opposite the right angle in triangle *PRQ*.

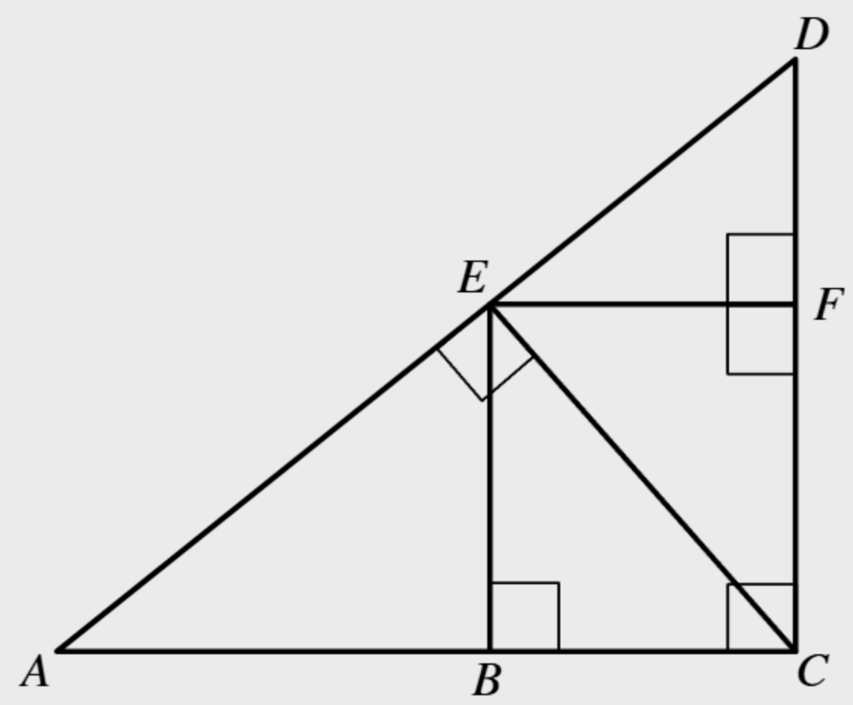
1.1 Intelligent practice: answers p.8

Support students to use the Reflect, Expect, Check, Explain model by using prompts: *Have you seen a question like this before? What has changed from the previous question part? How do you think that will affect the answer? Can you explain why you got that answer?*

1. **a** True **c** True **e** True
   b True **d** False **f** False
   g False (The angle measures a turn and the side is a measured length. These cannot be compared.)
2. **a** 12 cm **b** 90° **c** 13 cm
3.

Diagram	$AB =$	$CB =$	$B\hat{A}C =$
	8 cm	17 cm	90°
	5 cm	5 cm	45°
	5 cm	5 cm	60°

4. **a** $\angle CAB = \angle ACB$
   b $\angle EFD = \angle EDF$
   c $\angle GIH = \angle IHG$
5. **a** *AC*
   b *EF*
   c *GH*
   d *LK*
6. **a** *QS*
   b *RS*
   c Triangle *PRS* has no hypotenuse because it is not a right-angled triangle.
7. **a** *AE*
   b *AC*
   c *CD*
   d *AD*
   e



In **Question 1**, students are presented with a triangle and asked to determine whether the mathematical statements in each part are true or false. Parts **a–c** vary direction. Part **d** is a good starting point to ask students working toward extending outcomes if they think this angle is actually 0°. Or could it be 360°? Part **e** changes one letter from part **d** to prompt reflection. Part **f** introduces comparison of sides when angles have been given to prompt discussion of the accuracy of sketches. Part **g** tries to compare the size of an angle with the size of a side. This is nonsensical.

Question 2 again describes notation and aspects of the diagram presented. In part **c**, students need to measure a line segment that is not presented on the page.

Question 3 requires students to copy and complete the table of properties. In the final row, there are no angles given. However, the equilateral triangle is shown by the dashes, so the equal lengths and equal angles can be found. You might need to remind some students that the angles in a triangle sum to 180°.

Questions 4 and **5** show closely related triangles in different orientations. For **Question 4**, the equal angles of an isosceles triangle are related to the dashed sides. In **Question 5**, the hypotenuse will be identified as the side opposite the right angle.

Question 6 develops this idea of the hypotenuse further by asking about the contexts in which a side is and is not a hypotenuse. The two triangles are linked by a common side, SQ. This side is the hypotenuse in part **a** but not in part **b**.

This idea becomes more complex in **Question 7**, where multiple triangles are joined together. You might need to remind some students that angles on a straight line sum to 180°.

1.1 Which method?: answers p.9

1

Shape	Type of polygon	Length of AB	Length of BC	Type of angle $A\hat{B}C$	Perimeter	Area
a	**Square**	**8 cm**	**8 cm**	**Right angle**	**32 cm**	**64 cm^2**
b	**Rectangle**	**8 cm**	**10 cm**	**Right angle**	**36 cm**	**80 cm^2**
c	**Rectangle**	**10 cm**	**8 cm**	**Right angle**	**36 cm**	**80 cm^2**
d	Right-angled triangle	10 cm	8 cm	Acute	24 cm	24 cm^2
e	**Parallelogram**	**8 cm**	**5 cm**	**Obtuse**	**26 cm**	**32 cm^2**
f	**Hexagon**	**10 cm**	**5 cm**	**Right angle**	**36 cm**	**65 cm^2**
g	**Isosceles trapezium**	**10 cm**	**5 cm**	**Acute**	**24 cm**	**28 cm^2**

2 **a** $\angle GFI = 60°$ **c** $\angle AIB = 30°$ **e** $\angle CIB = 30°$ **g** $\angle ECI = 40°$
b $\angle GIF = 30°$ **d** $\angle ABI = 60°$ **f** $\angle BCI = 75°$ **h** $\angle CEI = 50°$

3 **a** **i** $w = 20$ **ii** $x = 50$ **iii** $y = 75$ **iv** $z = 5$
b **i** $G\hat{F}H = 65°$ **ii** $\angle GFI = 95°$ **iii** $\angle NMP = 75°$ **iv** $N\hat{M}Q = 75°$
c PR and MS are parallel.

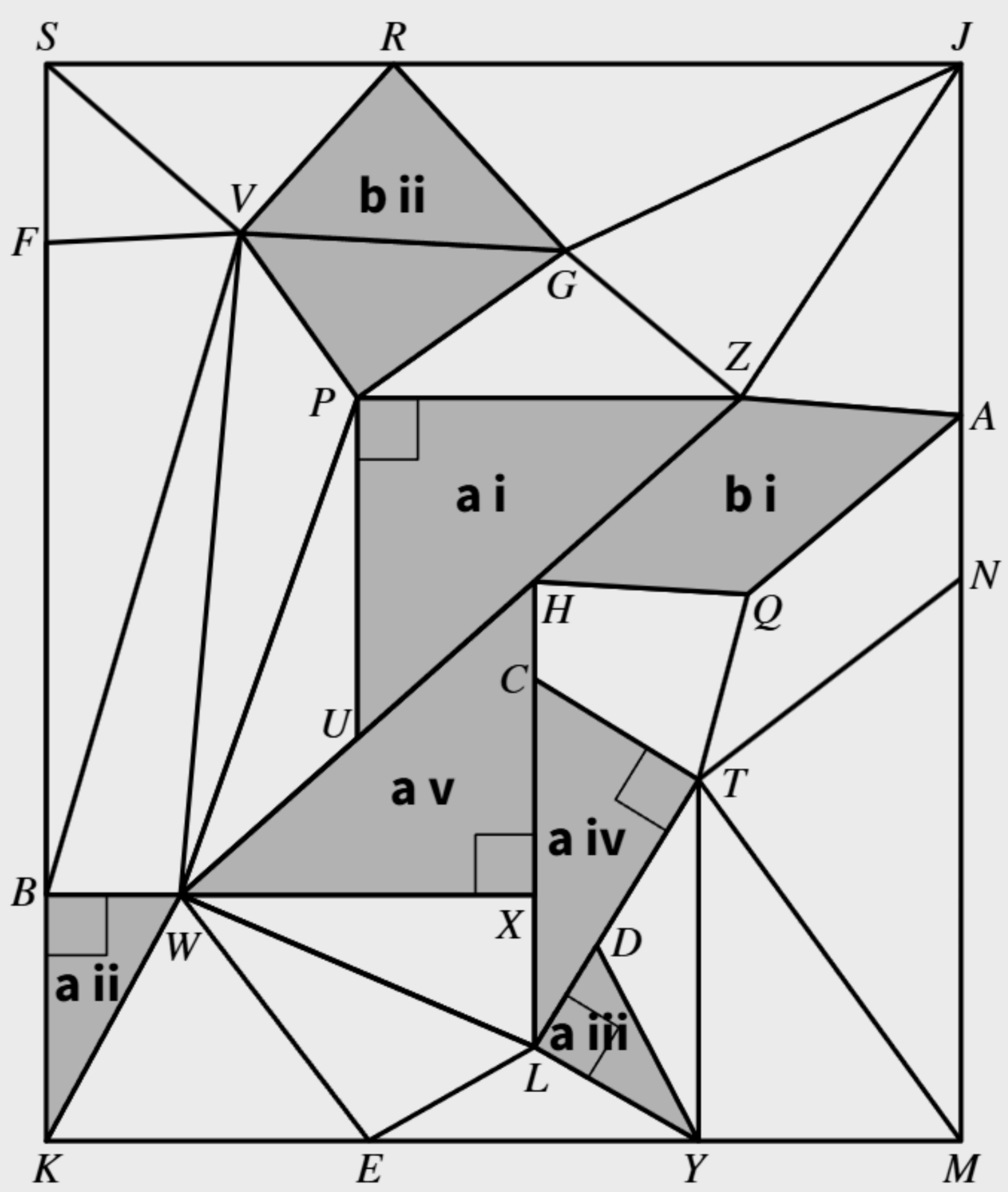
Question 1 links to **Question 5** from the Intelligent practice section. Here, however, there are explicit links to names of quadrilaterals, types of angles, areas, and perimeters. Parts **b** and **c** have the same measurements but the vertices are labelled differently. In part **f**, there is explicit linking with parallel line notation. This tells us the angles are all either 90° or 270°.

You could give students a copy of the diagram for **Question 2**. Students should label the known angles on the diagram as a first step. Students might have difficulties with vertically opposite angles around point I and identifying that line segment HI and line segment IB lie on the line segment HB. Students who have difficulty with finding specific angles could find any angle they are able to first and then return to the actual questions after this.

In **Question 3**, students have to interpret a complex diagram that uses angle notation and they have to form and solve equations. You could give students a copy of the diagram. For part **a i**, the half turn of line segment ABC will set up the equation to find w. In part **a ii**, x can be found by forming an equation using line segment BDF. For part **a iii**, y can be found using line segment BJK. Students do not have enough information to use line segment DFI, as angle GFH is unknown. In part **a iv**, z can be found by using line segment QPM. In part **b**, students need to understand different forms of angle notation and use angle sums in half turns. In part **c**, students need to remember their knowledge about angles in parallel lines. If students find this difficult, suggest that they label their diagram with all of the angles they have found and look for equal corresponding angles.

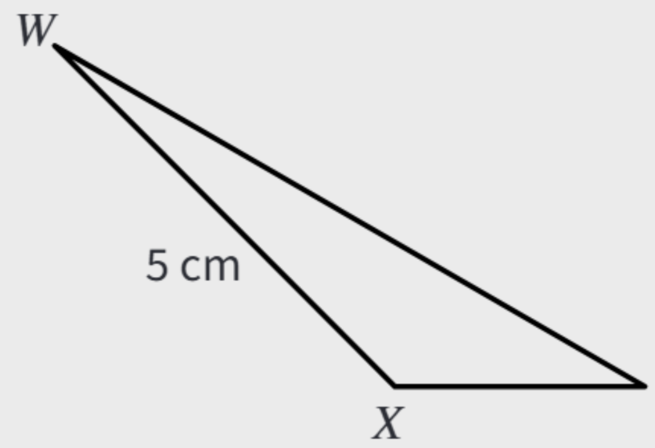
1.1 Expert practice: answers p.11

1

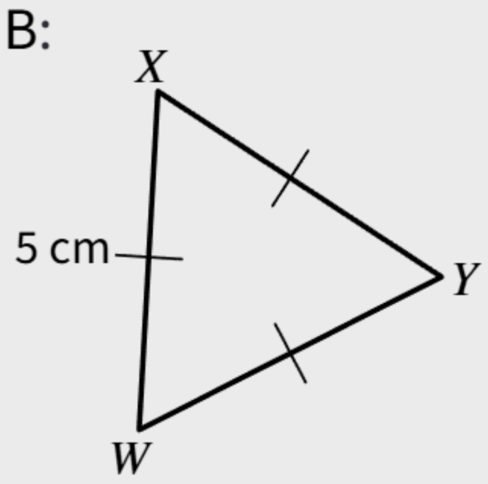


2 Example answers:

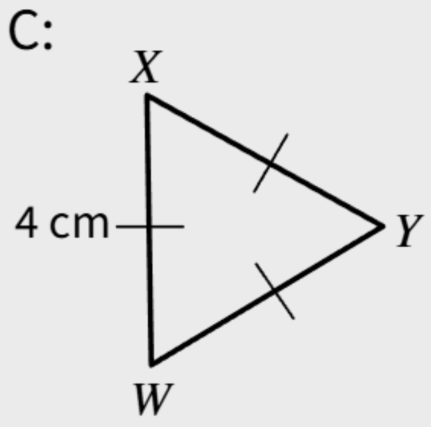
A:



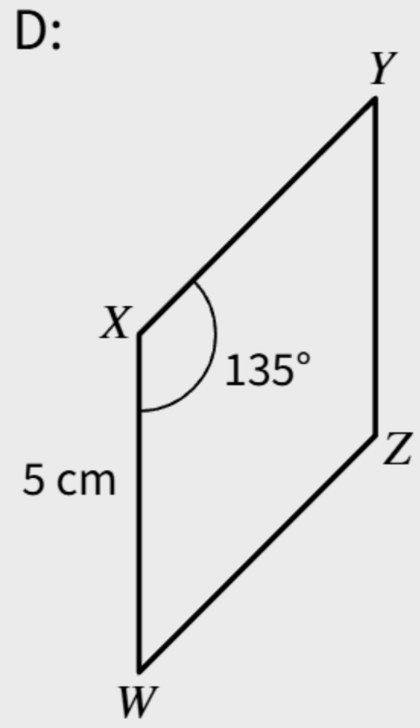
B:



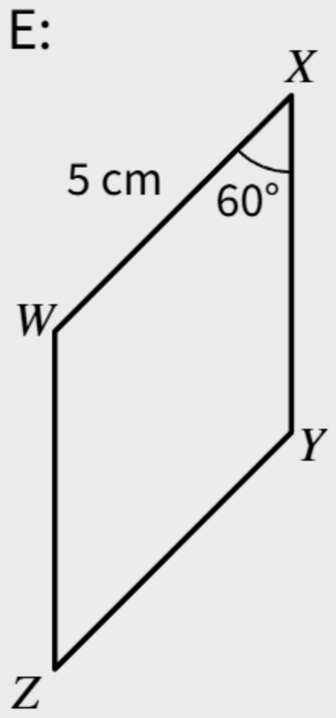
C:



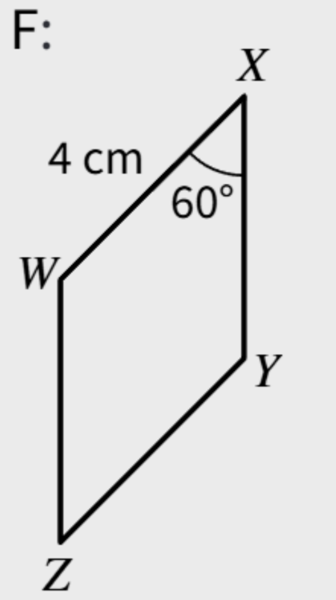
D:



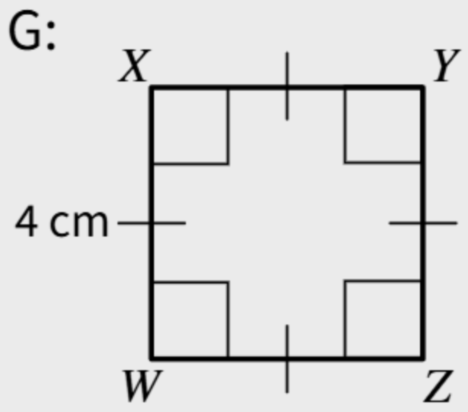
E:



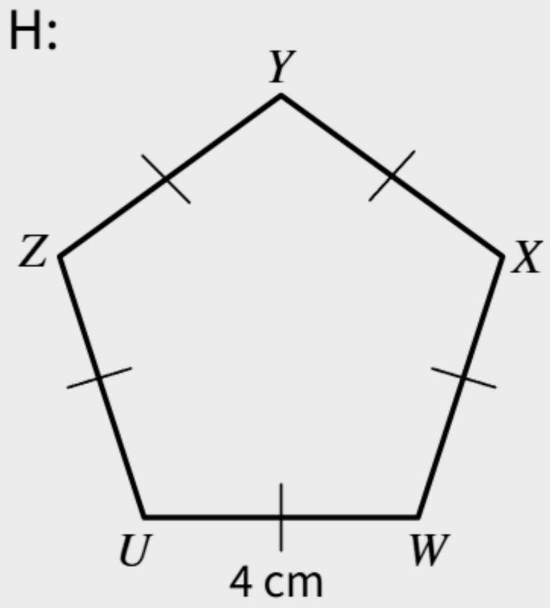
F:



G:

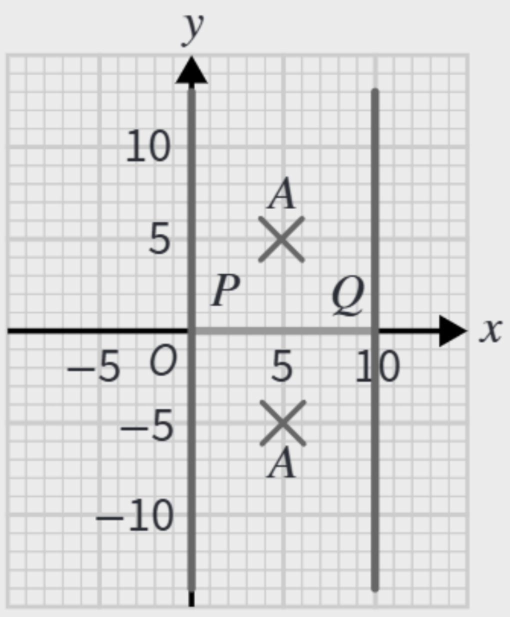


H:



3 **a** **i** Students' answers

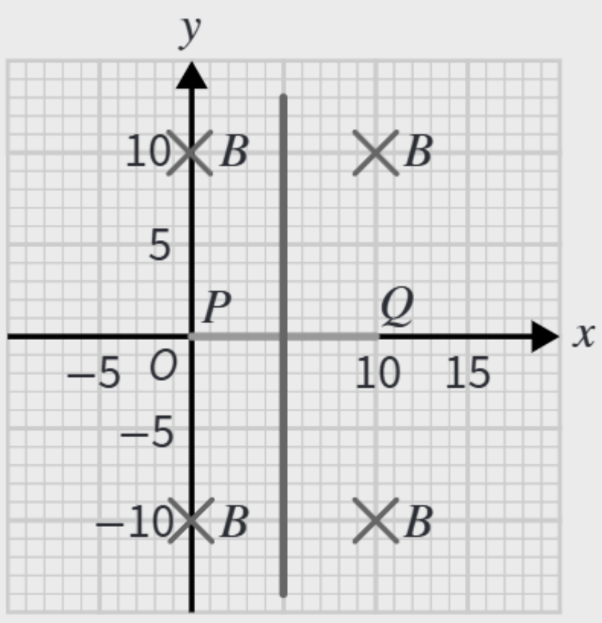
ii



A can be any point on the line $x = 10$, e.g. (10, 5), any point on the y-axis, e.g. (0, 10), or either of the points labelled *A*.

b **i** Students' answers

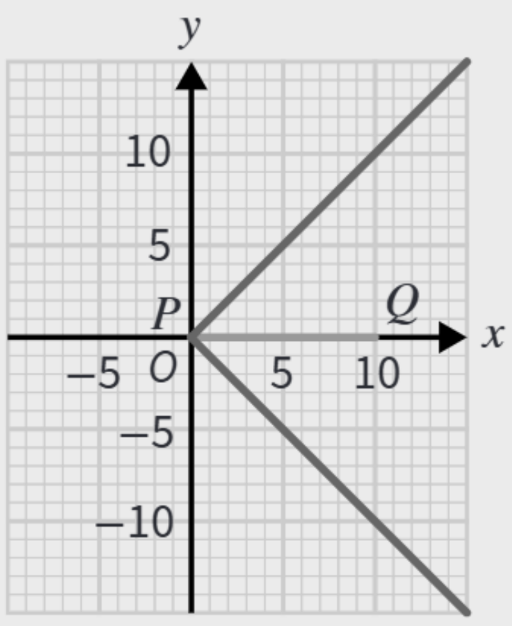
ii



B can be any point on the line $x = 5$, e.g. (5, 5) or any of the points labelled *B*.

c **i** Students' answers

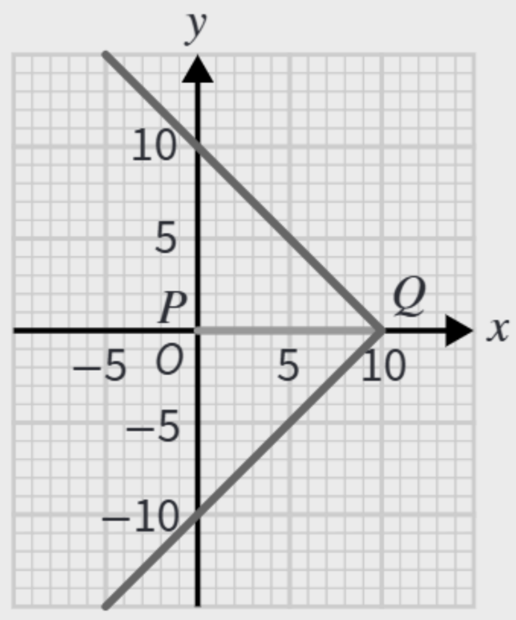
ii



C can be any point on the line $y = x$, where $x > 0$, e.g. (5, 5), or any point on the line $y = -x$, where $x > 0$, e.g. (5, –5).

d **i** Students' answers

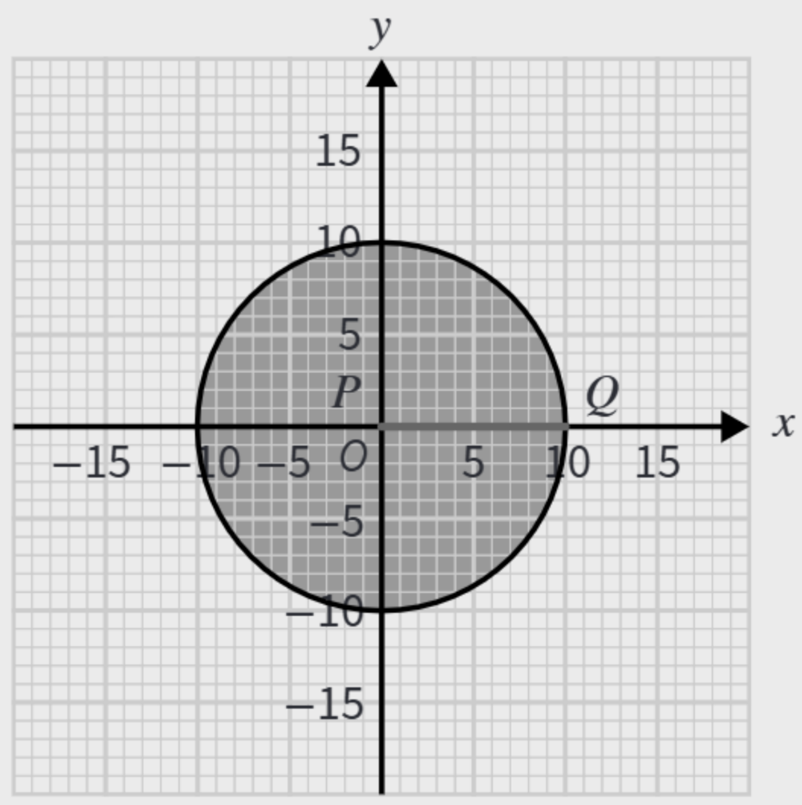
ii



D can be any point on the line $y = 10 - x$, where $x < 10$, e.g (0, 10), or any point on the line $y = x - 10$, where $x < 10$, e.g. (0, –10).

e **i** Students' answers

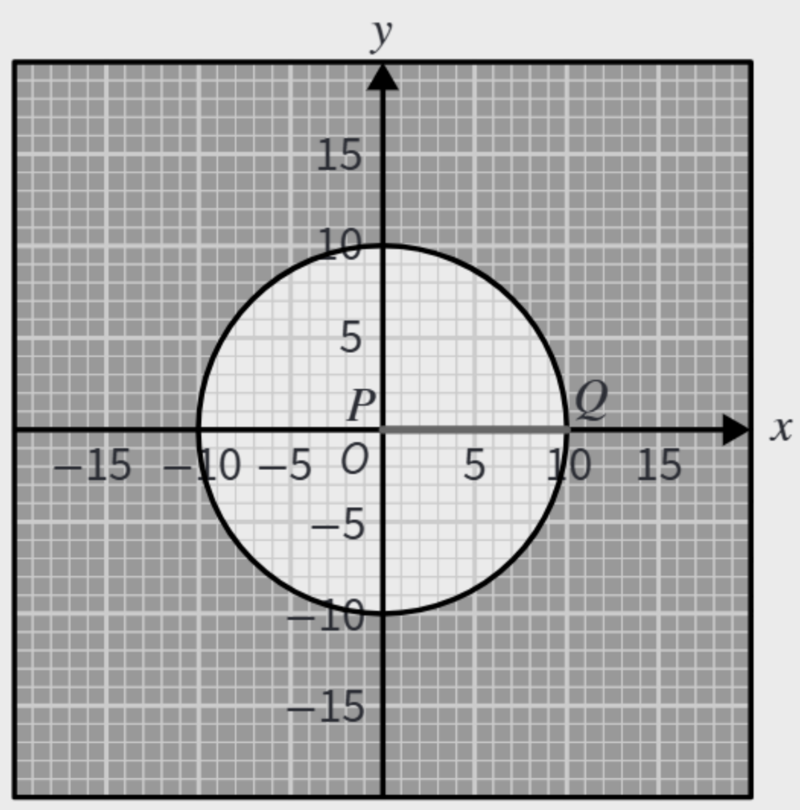
ii



E can be any point inside a circle centre (0, 0) with radius 10 units.

f **i** Students' answers

ii



E can be any point outside a circle centre (0, 0) with radius 10 units.

Give students a copy of the diagram for **Question 1**. They should use the naming conventions to shade in the relevant shapes. This should be accessible for students working on developing outcomes as well as students working towards secure and extending outcomes. To make this task more open-ended, provide additional copies of the diagram and ask students to shade in a selection of shapes. They could work in pairs and, hiding their answers from their partner, they could communicate which polygons require shading.

Question 2 gives students a Venn diagram to complete. For students at a developing level, you could sketch a series of simple triangles/quadrilaterals with labelled vertices and angles and ask them to categorize these triangles/quadrilaterals. For students working towards extending outcomes, increase the level of difficulty by adding an extra condition. For example, the area of every shape in the Venn diagram must be $10\,cm^2$.

In **Question 3**, students link the formal notation to the coordinate plane. At the developing stage, students can access this question by checking some points one by one or by finding points that definitely do not work for the context. Prompt students working towards extending outcomes to find all points that satisfy the given conditions by sketching diagrams.

1.2 Similarity

Now that students have formalized notation conventions of shapes, this section explicitly integrates work from two previous chapters (multiplicative relationships from Student Book 7, Chapter 8 and transformations from Student Book 7, Chapter 9). As such, this section focuses equally on what similarity is as well as on what similarity is not.

	Learning outcomes		
Learning objectives	**Developing**	**Secure**	**Extending**
Identify similar shapes	Recall that 'similar' means that shapes have sides in the same proportion to each other and the same angle sizes, but the lengths of the sides can be different Identify examples of similar shapes *e.g. Find the pairs of similar shapes from a diagram of 10 polygons, not triangles, including two types of rectangle, a pair of squares, and a reflected rectilinear shape.*	Know that enlargements, rotations, translations, and reflections produce similar shapes *e.g. Shapes A and B are similar. State how you could transform shape A to shape B.* Identify non-examples of similar shapes with proportional sides but angles in a different order, and shapes with equal angles but where side changes are additive	Recognize all circles as being similar even though they are not 'angled' or polygonal, and recognize similarity in some 3D shapes *e.g. Are all cubes similar? Are all cuboids similar? Explain your answer.*
Find missing angles in similar shapes	Find missing angles in pairs of similar triangles where they are drawn in the same orientation *e.g. Triangles ABC and DEF are similar. Find angle DEF.*	Find missing angles in pairs of similar triangles and quadrilaterals where the complete information is only given by using the property of preserved angles	Find missing angles in pairs of similar shapes where these are drawn with a common side or with a common vertex with one shape inside the other
Find missing lengths in similar shapes	Find missing lengths in simple similar shapes where the scale factor is a positive integer and the shapes are drawn in the same orientation *e.g. The two rectangles are similar. Find the length PQ.*	Find missing lengths in similar shapes where the scale factor is a positive integer or a unit fraction and recognize the proportionality relationship between these as reciprocals	Find missing lengths in similar shapes where the scale factor is fractional and recognize the proportionality relationship between these as reciprocals