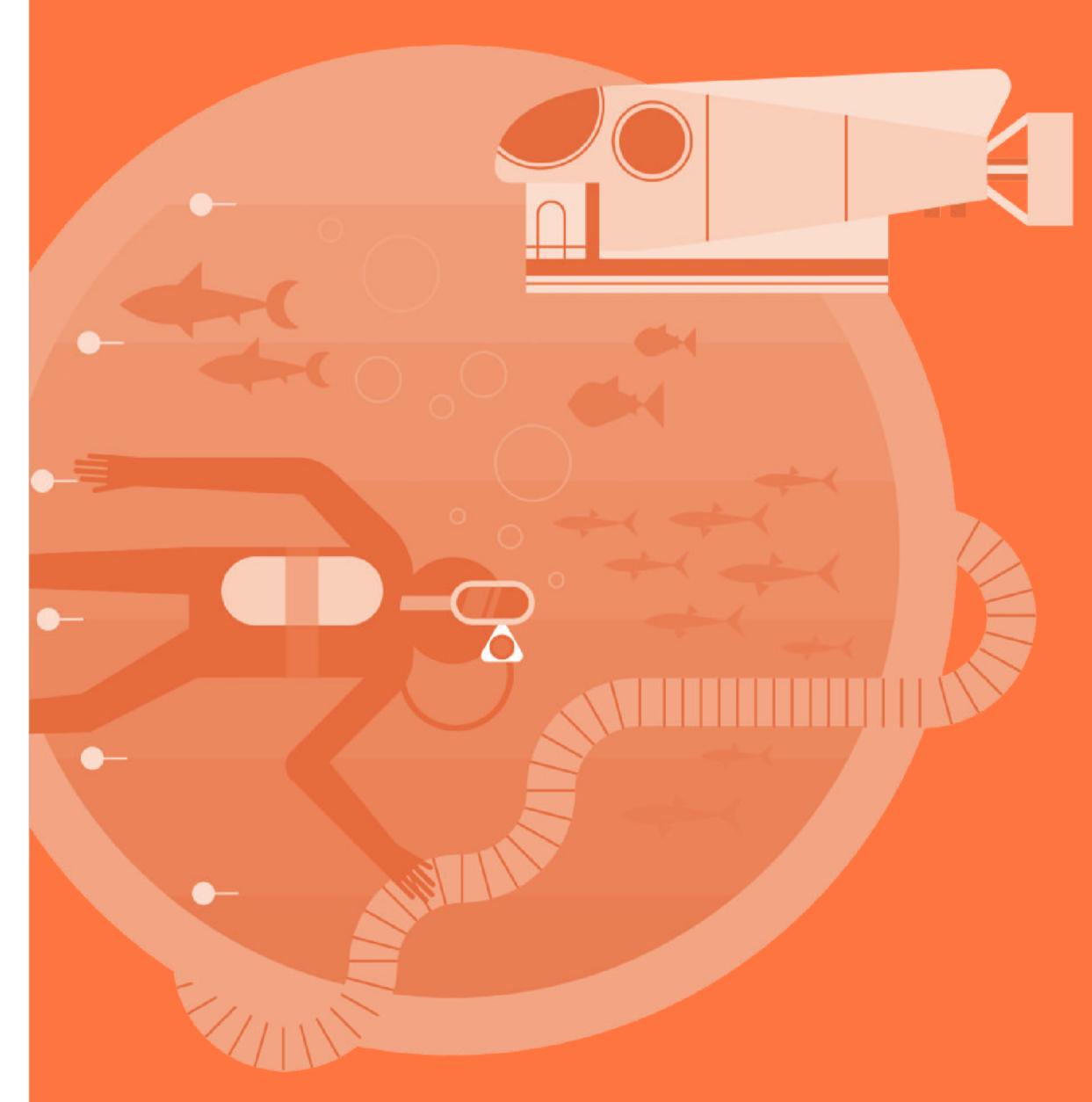




Maths

Teacher's Guide

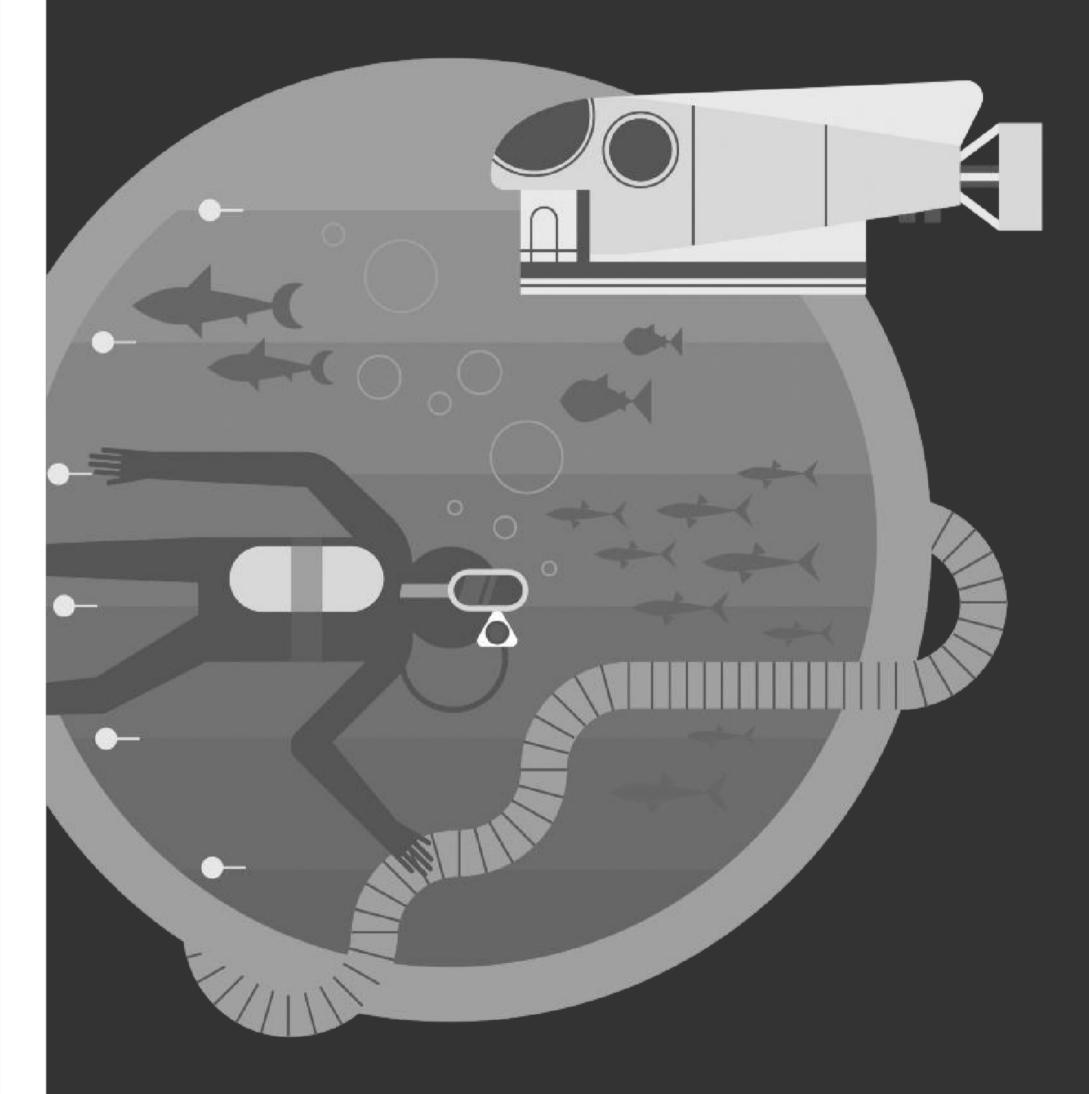


Lower Secondary



Maths

Teacher's Guide



Craig Barton

Charlotte Hawthorne
Dan Draper
Helen Konstantine

Jemma Sherwood Katie Wood

OXFORD



Great Clarendon Street, Oxford, OX2 6DP, United Kingdom

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Introduction

The joy of learning maths

We are living in an ever-changing world, where the way we work, live, learn, communicate, and relate to one another is constantly shifting. In this climate, we need to instil in our learners the skills to equip them for every eventuality so they are able to overcome challenges, adapt to change, and have the best chance of success. To do this, we need to evolve beyond traditional teaching approaches and foster an environment where students can start to build lifelong learning skills. Students need to learn how to learn, problem-solve, be agile, and work flexibly. Going hand in hand with this is the development of self-awareness and mindfulness through the promotion of wellbeing to ensure that students learn the socio-emotional skills to succeed.

Teaching and learning with Oxford International Maths

This course is suitable for use alongside the Oxford International Curriculum and the English National Curriculum. The books for each year (or stage) follow the scheme and meet the learning objectives for both curricula. Objectives are written in student-friendly language in the Student Book.

The Lower Secondary course is designed for students aged 11 to 14. Each year has a Student Book and a Teacher's Guide. There are also numerous digital resources and sources of support on www.kerboodle.com. These three core elements provide a cohesive offer that supports learners to develop and consolidate knowledge and skills, and to draw connections between topics.

Oxford International Maths at Lower Secondary is a mastery course. The term 'mastery' has been drawn from teaching approaches in countries where maths performance is high. The essence of mastery is to produce students who have deep conceptual understanding and procedural fluency through learning in a collaborative and problem-solving context. Mastery learning incorporates exposure to different methods of solving a problem, mathematical dialogue, and explanation.

This course has six main aims:

1 Provide clear and coherent curriculum pathways

Close attention is paid to the sequencing of concepts, connections with prior learning, common difficulties and misconceptions, and consistent use of key models and language.

Pre-requisite, current, and future learning are signposted to ensure that learners are clear where they are on their learning journey. Checks and diagnostic questions help teachers identify where learners are before teaching a new topic, and integrated assessment provides regular feedback on learners' progress against the curriculum pathway.

2 Hold high expectations, aspirations, and ambitions for all learners

Developing, secure, and extending (DSE) learning outcomes underpin each learning objective to ensure that the same curriculum is accessible to all, with the aim that all learners achieve a secure level.

Support for reactivating prior learning and identifying misconceptions at the start of each topic helps reduce the attainment gap and ensures that all learners are at a suitable starting point for the new content. Carefully graduated practice allows all learners to build confidence while also providing stretch for those who are ready for more challenge. Differentiation is supported through questions targeted at each of the DSE outcomes.

3 Support engaged, self-regulated, and metacognitive learning

The Reflect, Expect, Check, Explain (RCE) framework has been developed by Craig Barton. It encourages metacognitive thinking where students think about their learning, feel empowered to reflect on their strengths and areas for development, and make connections across topics.

4 Promote development of learner identity and identification with each subject

The relevance of maths to learners is highlighted throughout curriculum components and resources, including student-facing facts and visuals that encourage thinking about real-world applications and provide prompts for discussion.

5 Enable responsive teaching and learning that continually evolves and improves

Resources are provided to help teachers check understanding at every stage in the learning process, and guidance about common difficulties and misconceptions supports teachers to respond to learners' needs.

Regular formative assessment on your Kerboodle subscription provides learners with personalized next steps to secure or extend their learning. Detailed reporting on Kerboodle supports both teachers and learners.

6 Inspire fascination and awe and wonder in the world around us

This course aims to stimulate fascination through a strong focus on thinking and working mathematically. Opportunities to develop the skills associated with working like a mathematician are embedded throughout.

Teaching techniques

Grouping students to promote a growth mindset

It is expected that you will use a variety of student groupings. It is important that you are active in deciding which form of grouping is appropriate, depending on the activity. In this way, students will learn how to work in a variety of ways and with a range of different students.

There are three main ways of grouping students:

- Friendship groups are most appropriate for activities in which students have been given some element of choice, for example, if they are carrying out some research for a data handling project. This grouping is the default if teachers do not actively group students.
- Ability groups, or groups based on students'
 prior experience, may be helpful if the lesson
 requires a very specific prior knowledge. You can
 group together the students that you know have
 this knowledge, and they can then work with
 minimal guidance from you. This will allow you to
 focus on groups who need additional support.
- Mixed-attainment groups are encouraged for the majority of activities. This form of grouping is also favoured by those following a mastery approach. Working in collaborative, all-attainment groups also supports students' wellbeing and promotes a growth mindset, as described in research by Carol Dweck. She found that students who were grouped by ability tended to stay in

those groupings throughout their school life, and regard themselves as having a fixed ability that could not be changed. This has dire consequences for students in middle or lower sets. When placed in mixed-ability groups, all students can develop a growth mindset that enables them to believe they can learn and improve, whatever their starting point (Dweck, C., 2007. 'The Perils and Promise of Praise'. Educational Leadership. October 2007, 65(2), 34–39). A growth mindset is promoted when students do not feel that their future success is predicated on prior achievement. This kind of grouping is particularly helpful for students new to English, as those who are less confident speaking the language will be able to hear their more confident peers using mathematical vocabulary. Research has shown that mixed-attainment groups benefit both high attainers, who become more secure in their maths knowledge through explaining their thinking to peers, and those less secure in their maths knowledge, through peer teaching.

Asking effective questions

The most skilled maths teachers can ask open questions to elicit students' current understanding. Skilful open questioning also allows students to articulate their current understanding carefully, and through this process either consolidate their understanding or come to realize where they have made a mistake. The following list offers a series of open questions that can be used whatever maths you are teaching.

- How are these the same/different?
- What would happen if ...?
- How else could you have done that?
- Why did you …?
- How did you ...?
- How do you know that is correct?

If you want students to check their solutions and consolidate their learning, ask them to explain how they reached their solution to a friend. Similarly, to support students in reflecting on their learning, you might ask the following:

- What maths did you use to solve the problem?
- What new maths did you learn?
- What key words did you use?
- What was the most challenging part of the activity?
- What did you do when you got stuck?
- What other questions could you ask?
- Did this remind you of any other areas of maths?

Differentiation

Differentiation is closely linked to inclusion: ensuring all students have access to the curriculum. This means that learning and teaching approaches must consider individual needs. Not all students will learn at the same pace or in the same ways.

This course supports the following approaches:

- Differentiation by task Content can be adjusted for some students to provide sufficient support or adequate challenge. The 'Fluency questions' in the Student Book are ramped, starting with questions aimed at less able students and finishing with 'stretch zone' questions. The latter are designed to extend more confident students and challenge them to think more deeply. 'Which method?' questions are also aimed at more confident students and require prior knowledge. 'Expert practice' questions are designed to be accessible to all, but self-differentiating depending on approach. For less able students, prioritize the questions in the Example-problem pair (EPP) grids.
- Differentiation by outcome This allows all students to tackle the same tasks, but with differentiated learning outcomes. There are three bands of differentiation for each learning objective: developing, secure, or extending. The differentiated outcomes are provided for each lesson in this Teacher's Guide. 'Secure' indicates that students have a secure grasp of the knowledge or skills specified. The band working towards 'secure' is 'developing', and the band moving past 'secure' is 'extending'.
- Differentiation by support This means providing more or less support as students are carrying out a task. Advice on this is provided

for each lesson in this Teacher's Guide. For additional practice, support sheets are available on Kerboodle to give less able students further opportunities to reach a secure understanding of new or challenging concepts in their own time. These worksheets can be tackled independently or used in adult-led, small-group sessions.

Assessment

Assessment is an essential part of learning. Without being able to check progress, teachers and students will not be able to identify areas of strength and areas in need of development.

Each activity – group and individual – can be assessed through observation, questioning, and progress notes. Written or drawn responses for each activity can be assessed/marked using the school's marking policy; and chapter, end-of-term, and end-of-year judgements made about individual and class progress.

Feedback is a crucial aspect of assessment.

This should be as positive and encouraging as possible, in which clear targets are identified.

Involve students in assessment and target setting – assessment is done with learners, not done to learners.

Formative assessment

This takes place during learning and is used to address issues as they arise. This means learning and teaching can be modified during lessons to better meet students' needs. Feedback is ongoing.

Each activity within the Student Book provides opportunities for formative assessment and

	Learning outcomes			
Learning objective	Developing	Secure	Extending	
Learners at this stage	are working towards secure knowledge and understanding but need more support to achieve this.	have a secure knowledge and understanding.	are working beyond expectations, and their knowledge and understanding can be stretched and challenged.	
e.g. Understand and list multiples of a number	List the multiples of a given number inside the times tables and use the term 'multiple' correctly	Understand multiples can be outside times tables up to 12	Understand that two multiples of a number add to give another multiple of the number	

feedback. You can do this by listening to explanations or paired discussions; observing students' workings; and assessing outcomes. Individual questions can be used to monitor understanding and identify misconceptions. These can be addressed as they are noted.

Summative assessment

This is used to measure or evaluate student progress at the end of a process – for example, when a chapter is completed or at the end of a year. Summative assessment compares students' attainment against a standard or benchmark.

The 'What have I learned?' pages at the end of each chapter can be used for summative assessment. You can record which questions each student is answering correctly and use this to measure individual attainment. It can also indicate how well the class is progressing through the work. In this way, the assessment can inform individual interventions (extra support for a student) or whole-class interventions (reviewing work that is not well understood).

How to support non-native English speakers

Ministries of Education at both local and national level are increasingly adopting the policy of English Medium Instruction (EMI), either for one or two subjects or across the whole curriculum.

In international schools, it is likely that students do not share a mother tongue with each other or perhaps the teacher. English is chosen as the medium of instruction to level the playing field and provide the opportunity to develop proficiency in an international language.

This does not mean that the maths teacher is expected to replace the English teacher, or to have the same skills or knowledge of English. However, they do need to become more language aware. This raises significant challenges, including:

- the teacher's knowledge of English
- students' level of English (which may vary considerably in international schools)
- resources that provide appropriate language support
- assessment tools that ensure that it is the content and not the language being tested
- differentiation that acknowledges different levels of proficiency in both language and content.

Language in the classroom

Using English in the classroom is very important as it provides exposure to an additional language (often a student's second or third), which plays a valuable role in language acquisition. The 'teacher talk' for purposes such as checking attendance and collecting homework does not have to be totally accurate or accessible to students. However, when teaching mathematical concepts, it is essential that the 'teacher talk' is comprehensible. The following strategies can help:

- simplify your language
- use short, simple sentences and project your voice
- paraphrase as necessary
- use visuals, the board, gestures, and body language to clarify meaning
- repeat as necessary
- plan before the lesson
- prepare clear instructions and check understanding.

Creating a language-rich environment

Providing a colourful and visually stimulating environment for students becomes even more important in the EMI classroom. Posters, lists of key words and structures, displays of students' work, and signs and notices in English all maximize students' exposure to English and, in big or small ways, contribute to their language acquisition.

Planning

In your planning, identify each language demand (LD). You will need to think about what language students will need to understand or produce, and decide how best to scaffold the learning to ensure that language does not become an obstacle to understanding the concept. This kind of language support (LS) goes beyond the familiar strategy of identifying key vocabulary.

Support for listening and reading

Listening and reading are receptive skills, requiring understanding rather than production of language.

If students need to listen to or read in English, ask yourself the following questions:

- 1 Do I need to teach any vocabulary before they listen/read?
- 2 How can I prepare them for the content of the text so that they are not listening 'cold'?

- **3** Can I provide visual support to help them understand the key content?
- **4** How many times should I ask them to listen/read?
- 5 What simple question can I set before they listen/read for the first time to focus their attention?
- 6 How can I check more detailed understanding of the text? Can I use a graphic organizer (e.g. tables, charts or diagrams) or gap-fill task?
- **7** Do I need to differentiate the task for developing and extending students?
- **8** Can I make the tasks interactive through groupwork or games?
- **9** How can I check their answers and give feedback?

Support for speaking and writing

Speaking and writing are productive skills and may need more language input from the teacher. You will need to think in detail about what language the task requires (language demands, LD) and what strategies you will use to help students use English to perform the task (language support, LS).

Ask yourself the following questions:

- 1 What vocabulary does the task require? (LD)
- 2 Do I need to teach this first? How? (LS)
- **3** What phrases/sentences will they need? Think about the language for learning maths (e.g. predicting and comparing). What structures do they need for these language functions? (LD)
- **4** While I am monitoring this task, is there any way I can provide further support to less confident students? (LS)
- **5** What language will students need to use at the feedback stage (e.g. when they present their task)? Do I need to scaffold this? (LD, LS)

Teaching vocabulary and structures

Vocabulary

Learning key vocabulary is central to EMI, and 'learning' means more than simply understanding the meaning. Knowing a word also involves being able to pronounce it accurately and use it appropriately. Aim to adopt the following strategies:

 Avoid writing a vocabulary list on the board at the start of a topic and 'explaining' it.

- The vocabulary should be introduced as and when it arises. This helps students associate the word or phrase with the concept and context.
- Record the vocabulary clearly on the board. Check your pronunciation and spelling.
- Give students a chance to say the word once they have understood it. The most efficient way to do this is through repetition drilling.
- Use visuals whenever possible to reinforce students' understanding of the word or mathematical concept.
- Advise students to record the vocabulary systematically in their glossaries under chapter or topic headings.
- Remember to recycle and revise the vocabulary.

Structures

Students will need to use phrases and sentence frames to discuss or write about their learning in maths, including these structures:

X is the same as Y.

The next number in the sequence is ... because ...

I predict that X will happen.

If X happens, then Y happens.

The next step is ...

Build up these banks of common maths phrases and remind students to record them. You do not have to focus on grammar here as the language can be taught as 'chunks' rather than specific grammatical structures.

Component overview

Student Books

The Student Books are textbooks for students to read and use. They include everything you need to deliver the course to your students, guide their activities, and assess their progress.

Student Book	Typical student age range
Student Book 7	Age 11–12
Student Book 8	Age 12–13
Student Book 9	Age 13–14

Teacher's Guides

There are three Teacher's Guides, corresponding to the three Student Books. Each Teacher's Guide includes:

 advice on delivering maths lessons effectively for EAL students

- a brief introduction to each chapter, including pre-requisite knowledge and mathematical concepts that will be revisited, an introductory activity using a picture prompt, teaching strategies, common learning misconceptions and real-world applications of maths
- guidance on teaching each Student Book topic, including student learning objectives and outcomes, recommended scaffolding, answer keys and approaches to problem-solving.

Digital

Kerboodle online learning (www.kerboodle.com) provides engaging digital books, lesson resources, and a comprehensive assessment package.

Digital books

- For the teacher: You can access the Student Books and Teacher's Guides as digital books. The digital books show the course content on screen, making it easier for you to deliver engaging lessons. A set of tools (e.g. sticky notes, bookmarks, pen features, zoom in, and spotlight text) is available to personalize your digital book and make notes. You can share your notes or hide them from view.
- For the students: Students can access the Student Books as digital books for use at home.

Resources

- Videos on each topic, also integrated into students' adaptive learning journey
- Exercise handouts useful visual aids and additional scaffolding for answering the Student Book questions
- Support worksheets and answer keys extra fluency questions for students who need more practice at developing-level questions
- Example-problem pairs worksheets additional practice and support for completing EPP grids
- Vocabulary quizzes for each chapter, to assess students' understanding of key terms
- Mapping to the English National Curriculum, Cambridge international curriculum, and Oxford International Curriculum
- Guidance on how the series supports progression to further study at iGCSE
- Letters to parents/carers to introduce the course and offer guidance on home learning.

Assessment and adaptive learning journey

With a Kerboodle login, you can access all the quizzes and tests. First, you will need to import class registers and create user accounts for your students. Once your classes are set up, you can assign students assessments to complete.

Our assessment model combines formative and summative practices. An additional element is regular, low-stakes quizzing aimed at helping students retain new concepts. The formative assessment comprises:

- My self-study quizzes at the end of each topic ask students questions that are relevant to the learning objectives they have just covered. Students' scores will generate either a 'developing to secure' next-step intervention, or a 'secure to extending' next-step intervention. The teacher will also see a breakdown of how students are performing against each of the learning objectives.
- Formative tests which cover content from the whole chapter. Similarly, students will be assigned a next-step intervention according to their score.

Quizzes and tests are auto-marked. Following either assessment type, students are offered personalized next steps. They can consolidate their knowledge if they are at a developing level, or challenge themselves if they have demonstrated secure knowledge.

At the end of each chapter, there is a paper-based summative assessment designed to evaluate understanding of the whole chapter.

Reporting and insights

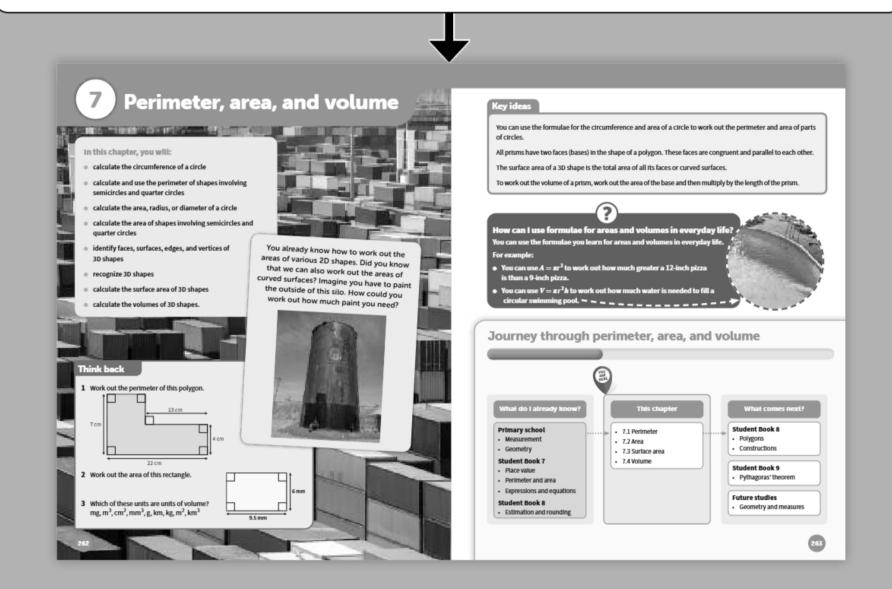
The formative assessment data will feed reporting on Kerboodle and give insights into strengths and areas for development. The data is broken down into learning objectives, and will support you in diagnosing learner needs and focusing your intervention accordingly.

Tour of a Student Book

Chapter opener

This explains to students what is coming up in the chapter. The 'Learning journey' map shows clearly what maths students should already know from previous learning, the new topics they will study in the chapter, as well as the next steps in their maths learning.

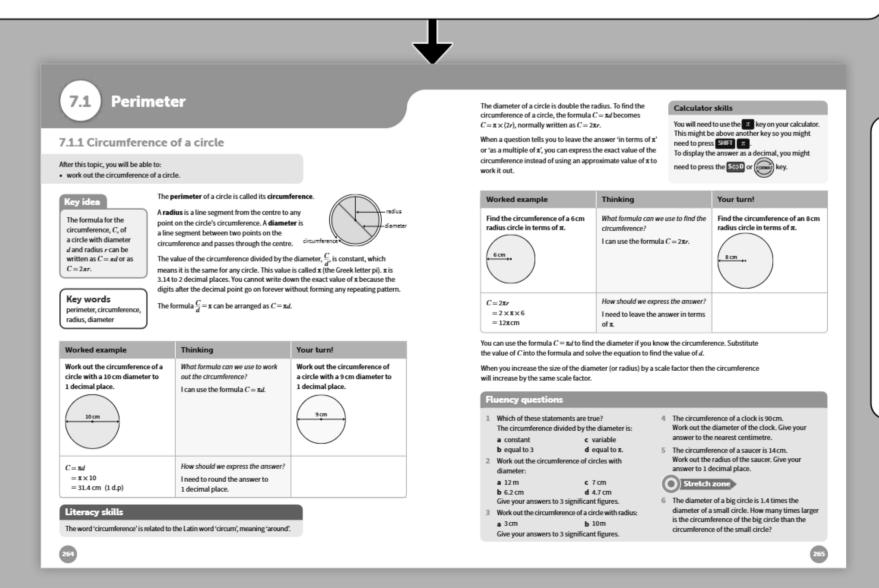
The 'Think back' questions help students recall existing knowledge. This feature will warm up their thinking and alert you to any gaps in their learning before carrying on.



Lesson pages

These pages guide students through a particular topic in each chapter. Simplified language is clear and accessible for English language learners, to ensure that a developing understanding of English does not get in the way of grasping key concepts. Skills boxes and fluency questions can then be used to check students' understanding of what they have just read and to stretch their thinking further.

Learning objectives for the lesson are clearly set out at the start and summarized in the Key idea box.



Key words boxes highlight the main maths vocabulary for the lesson. These words are also found in the Student Book glossary.

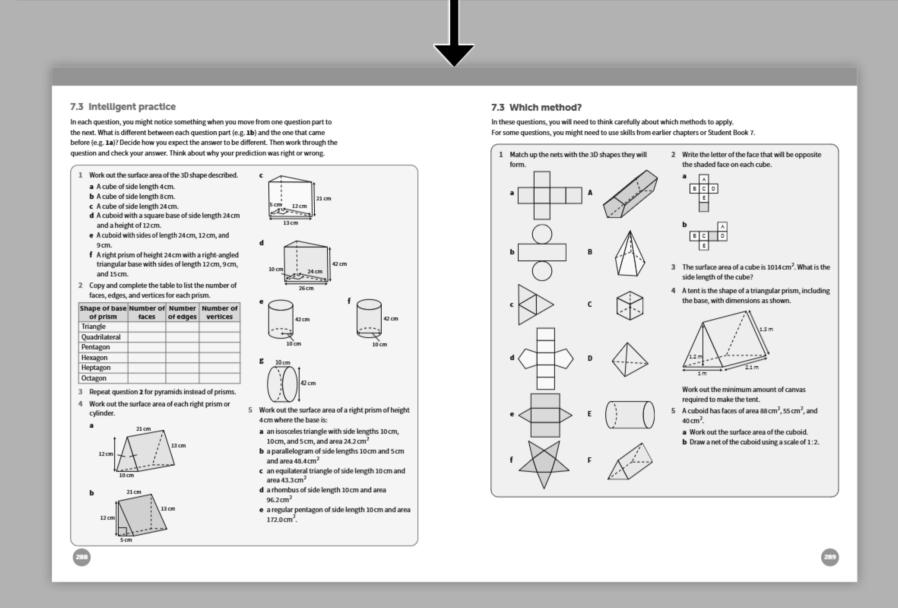
Intelligent practice, Which method?, Expert practice

After the lesson pages for each topic, there are three different types of exercise for students to apply and practise the maths they have just learned:

- 1 Intelligent practice
- 2 Which method?
- 3 Expert practice

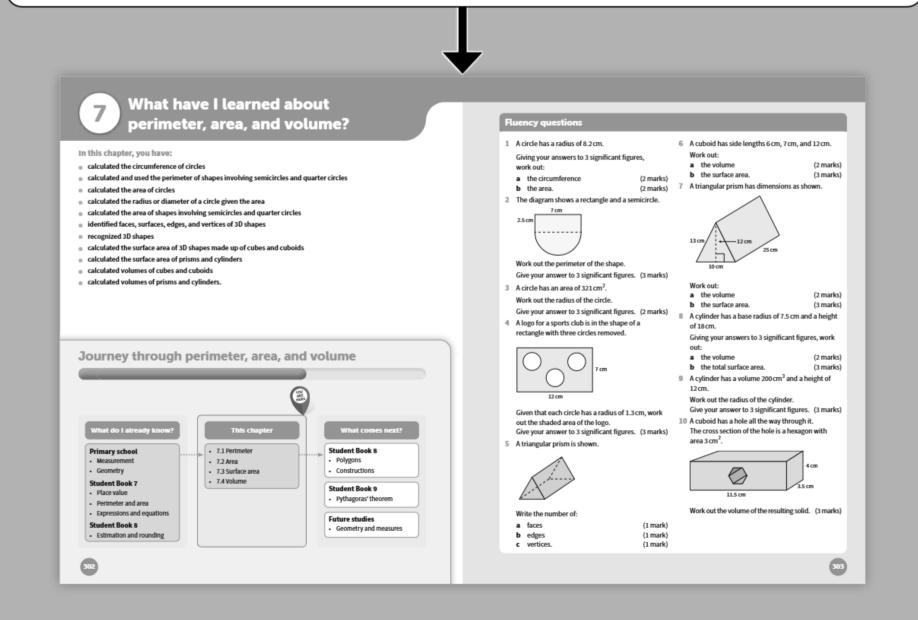
Each exercise works in a particular way to help the brain make connections, remember the topic, and recognize when to use it.

Find more information about these exercises on p.xiii of this Teacher's Guide.



What have I learned? pages

These pages summarize the content that students have learned so far and show how they have progressed in their learning journey. Each chapter concludes with exam-style questions to test how well students have learned and understood the topics, and to keep track of their overall progress.



The Learning Episode

Introduction

To stimulate interest in a way that helps students feel confident that they can be successful in the maths that will follow.

· Link to the big picture

The big picture in the chapter opener aims to provide both a purpose to students' study and a platform upon which to build strong memories (e.g. visual examples).

· Tell a story

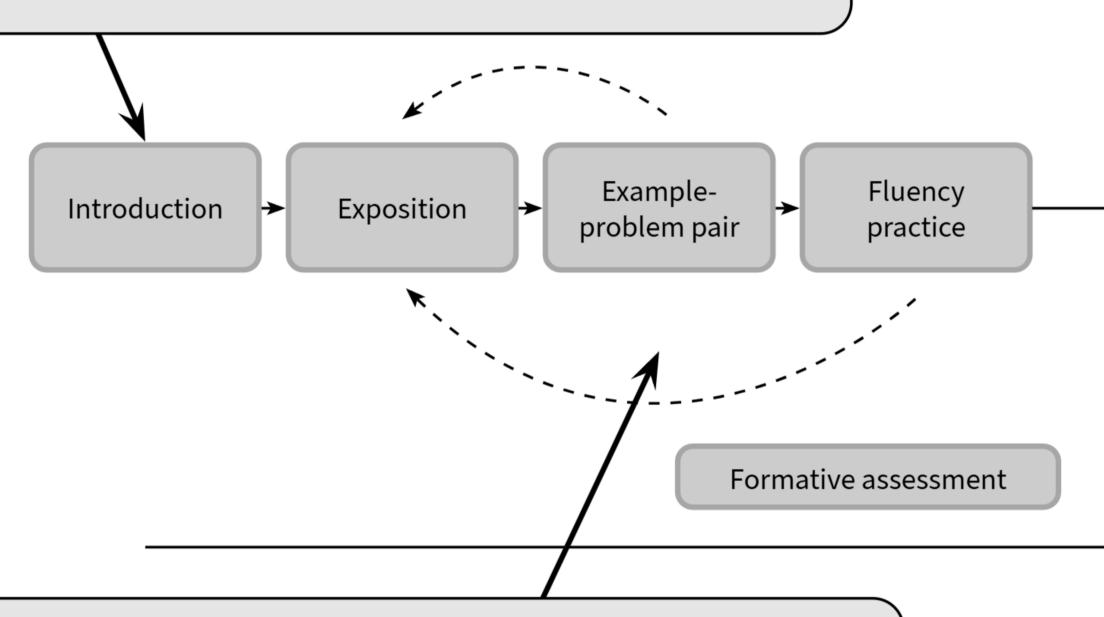
This will make the information easier to comprehend and remember (e.g. personal reflection – memories of learning it as a child or how you used the knowledge recently).

· Provide a question hook

This should be short and snappy, to spark student interest. You can use the questions linked to the big picture or real-world application in the chapter opener.

Discuss the etymology of key words

This will make it easier to remember the words' meaning, showing students how words connect to other areas of maths and the world. You can use the Literacy skills boxes throughout the Student Book.



Example-problem pair

To model how to do a new method in a way that allows students to be actively involved and attend critical elements.

1 Silent teacher

Run through the example without verbalizing any questions, but pausing and turning to students to cue them to think about what has happened.

2 Narration

Go through the example again, this time using thinking prompts to draw students' attention to critical elements of the working out.

3 Read the maths

Students use the thinking prompts to add annotations to the example (downloadable worksheets are available on Kerboodle) so that it makes sense.

4 Your turn

Students complete the question using the model example to support them with answering the question.

5 Share learning

Use a visualizer and examples of student work to highlight examples of good practice, misconceptions, etc.

Intelligent practice

These are sequences of questions that enable students to gain practice in carrying out a mathematical method, while providing opportunities to think mathematically.

1 Model the relationship

Model the Reflect, Expect, Check, Explain process first using an example question.

2 Silent practice

Students complete questions in silence, allowing them to think before they ask for help and make connections at the points at which they are ready.

3 Paired discussion

Students rehearse and modify their explanations, and listen and learn from others.

4 Discuss relationships

Reveal the answers and delve deeper into one or two relationships.

5 Prompt to delve deeper

Provide additional prompts to challenge the class or individuals.

Expert practice

These questions are less structured than in the Intelligent practice exercise. They provide practice of a key method, while providing opportunities for students to think more deeply and with greater purpose.

Five principles:

- 1 Students need to experience early success to support motivation.
- 2 There must be plenty of opportunities to practise the key procedure.
- 3 The practice should feel different to prevent boredom and allow students to see things from a different angle.
- 4 Opportunities must exist for students to make connections, solve problems, and think more deeply.
- 5 The focus is always on the practice, allowing students to make connections at their own pace.

Goal-free problems Watch out! Intelligent Which **Expert** practice method? practice Review exercise **Future** learning episodes

Goal-free problems

Working on these types of problem is more likely to secure the understanding of concepts and connections due to reduced pressure on working memory. Goalfree problem exercises are provided at the end of each chapter in this Teacher's Guide.

Watch out!

This exercise introduces students to common misconceptions in the topic areas they have just been studying, at a point when their understanding is secure enough to understand them. This activity also supports students' reasoning skills. They are provided as downloadable worksheets on Kerboodle.

Review exercise

These end-of-chapter fluency questions, with mark scheme, encourage students to reflect on their learning and test themselves to see if they have understood the content.

Which method?

This exercise ensures that students know when and when not to apply a given method. It also provides retrieval opportunities for previous learning. These questions:

- include ideas from the current Learning Episode along with ideas from previously covered Learning Episodes, which on the surface appear similar
- support students in identifying the strategy needed to answer a question
- · highlight the importance of reading the question
- allow students to appreciate exactly what it is about a question that requires a given method.
- **1** Model how to approach a question and work out what method is needed.
- **2** The *Reflect, Expect, Check, Explain* process could be used to support students' thinking here. They can reflect on what is familiar about the question and what links they can see to other questions or topics, make an expectation about what method is needed, try the question and check their answer, and then reflect on whether that was the correct method and how they might approach similar questions in future.

Reflect, Expect, Check, Explain (RECE)

The Reflect, Expect, Check, Explain (RECE) model is a pedagogy that encourages student self-reflection when answering questions. RECE exploits the Self-Explanation Effect, which shows how inviting students to narrate their thought processes while completing tasks can improve understanding and long-term recall. In maths, it is a powerful way to form connections between concepts and help students reflect on how what they already know can help them understand what they do not.

Following the Reflect, Expect, Check, Explain prompts is particularly impactful when students are practising questions, where only one or two elements change between questions. RECE supports students to **reflect** on what they know and on the basis of this make predictions about what they would **expect** to happen. After completing the question, students **check** their answer and then attempt to **explain** why the answer is, or is not, what they expected.

The RECE model can be used in a variety of ways across a Learning Episode and this Teacher's Guide will prompt you where it is especially useful – for example, during direct modelling as part of an EPP-fluency cycle or Intelligent practice. Using RECE promotes more general reflective thinking, which has benefits throughout students' learning journey and ultimately supports them to become more confident mathematicians.

The RECE model has been designed to be simple to use, but like any new approach it is important to model first to students. This ensures that they understand how it works and can try it out themselves with support. Here are some prompts that you can use with students when introducing RECE:

- First, read the question. What do you notice?
 Based on previous questions that are similar,
 what has changed and what is the same?
- What would you expect to be the answer to this question? What do you need to do next that is the same and different? Will the answer increase/ decrease? etc.
- If the answer surprised you, can you explain why? If it did not surprise you (you answered correctly), how could you explain what you did to someone who does not understand yet? If you did not notice the link between the questions or explain the relationship before, can you do so now?

Addressing misconceptions

The Education Endowment Foundation (EEF) defines a misconception as 'an understanding that leads to a systematic pattern of errors'. That is, misconceptions can arise when a generalization has been applied outside of the context in which it is useful.

It is important that teachers have knowledge of possible misconceptions so that they can plan to prevent them before they arise, as well as uncovering and addressing them when they do.

At the start of each chapter in this Teacher's Guide, there is information about the common difficulties and misconceptions that may arise during teaching, with suggestions of how to avoid them. These are also discussed in context during the exposition of each Learning Episode.

Some things to consider when planning to address misconceptions:

- Early in the Learning Episode, focus only on the misconceptions that you **know** students hold.
- Consider how a misconception might have developed. What generalizations has the student misapplied and what counter examples could you use to challenge their belief?
- Plan opportunities for discussion, either as a class or in small groups.
- Use examples, questions, models, and images that allow students to make accurate generalizations and conclusions.
- Compare examples and non-examples of a concept, as well as standard and non-standard representations.

1 Estimation and rounding

Introduction to chapter

In this chapter, students revisit how to round numbers to the nearest integer, ten, hundred, thousand, and so on, before looking at rounding to a given number of decimal places.

Students then apply this knowledge to the principle of rounding to significant figures by identifying the place value of significant figures in a number and rounding accordingly. Finally, students apply their knowledge of rounding to the concept of estimating solutions to calculations. Estimating calculations is an important mathematical skill. Allow students to check the validity of answers on their calculators.

Core concepts

- Real numbers
- The base-10 numeration system
- Comparison
- Estimation

What have students already learned?

- Understanding the place value of digits in integers
- Understanding the place value of digits in decimals
- Knowing and converting between units of measure
- Calculating using the priority of operations
- Comparing and ordering fractions

What will students revisit in this chapter?

- Rounding any integer to a required degree of accuracy
- Rounding decimals with two decimal places to the nearest integer and to one decimal place
- Solving problems that require answers to be rounded to specified degrees of accuracy
- Developing skills of rounding and estimating as a way to predict and check the order of magnitude of answers to decimal calculations

Direct students' attention to the inset photo in the Student Book and encourage a discussion:

- Did the headline writers make a mistake by using \$20 million? (No, they chose to give an approximate number that is quicker to read than the more precise total of \$20309747.)
- A headline stating that the total raised was \$20108453 would be closer to the true value, but would be more misleading than using \$20 million. Why do you think this is? (Stating such a precise figure suggests that \$20108453 is the accurate total, which is not the case. Stating that the total is \$20 million suggests only that the total is approximately \$20 million.)
- In a 100 metre race at an athletics event, the winning time was 10.87 seconds and the slowest time was 11.41 seconds. Why might news reporters choose to give all these times in seconds to two decimal places, rather than approximate the times to the nearest second? (By approximating, or rounding, to the nearest second, all the finishing times in this case would be given as 11 seconds. If the times are reported to the nearest hundredth of a second, however, we can calculate differences between competitors with more precision.)

Teaching strategy

Students sometimes struggle with rounding 'to the nearest...' when dealing with decimals, more so than with integers. This may be because they are less familiar with decimals and less confident with decimal place value. Number lines, especially zoomable number lines found online, can be used to help them to position a number between two other numbers and see which is closest.

Common learning misconceptions

Common mistakes when working with significant figures include not putting placeholders in to keep digits in the correct columns (for example, when rounding 26 500 to two significant figures, writing 27 instead of 27000). Place-value tables can help if this is a problem. The other common misconception students have is that 'all zeros are not significant'. This means that when rounding a number such as 0.2034 to two significant figures (0.20), they write

0.203 (thinking that only the 2 and 3 are significant). It is sometimes the case that students do not like to end a number in a 0, because they have previously been taught that 0.20 = 0.2. This can lead to queries when the answer to 'Round 0.203 to two significant figures' is 0.20. It is important to emphasize the difference between zeros that are placeholders and zeros that are significant.

Broader context

The skill of estimation is essential for working effectively with numbers. The ability to approximate numbers appropriately helps us simplify problems so that we can think about them more easily.

The early twentieth-century physicist Enrico Fermi applied some of the techniques in this chapter to

estimate the strength of an atomic bomb before such a thing could be measured. Nowadays, scientists and engineers apply 'Fermi' estimates to get an idea of the value of extreme calculations before having to employ technical, complex, or expensive measures to obtain accuracy.

Rounding to decimal places

Students build on their existing knowledge of rounding to the nearest 10, 100, or 1000 from primary school, along with their knowledge of place value from Student Book 7, Chapter 1. This learning is revisited and then extended to rounding to greater powers of 10 by applying the same principles, and then to smaller powers of 10 by rounding to a given number of decimal places.

	Learning outcomes			
Learning objectives	Developing	Secure	Extending	
Round integers to a given power of 10	Round numbers to the nearest power of 10 using a number line E.g. Use a number line to round these numbers to the nearest 10: 69, 60, 9665, 42567982	Round numbers to the nearest specified power of 10 E.g. Round these numbers to the nearest 1000: 42567982; 4455; 1987.24	Identify the minimum and maximum for an integer rounded to the nearest specified power of 10 E.g. A number is given as 4000 rounded to the nearest 100. Work out the smallest number this could be.	
Round numbers to any number of decimal places	Round numbers to three decimal places, rounding up a third place digit if the next digit is 5 or more E.g. Round 0.0283 to 3 d.p. Using pre-drawn number lines to help, round numbers to any number of decimal places	Round numbers to any number of decimal places, rounding up the last required digit if the next digit is 5 or more	Identify a number that could have been rounded to the given decimal E.g. A number is rounded to 0.456 to 3 d.p. Which of these numbers could it be? 8.4562; 0.045 55; 0.456 49; 0.4556	

Tier 2 vocabulary	Tier 3 vocabulary
halfway, round, rounded	approximate, decimal place, degree of accuracy

Classroom resources

Models

• number lines

Digital resources



My self-study quiz, Example-problem pairs, Exercise handout, Extra fluency questions, videos

1.1.1 Rounding integers to the nearest 10, 100, 1000, and higher p.4

Objective

Students will learn how to:

• round integers to a given power of 10.

Students should be familiar with rounding from primary school. Use this section to revisit their existing understanding and deepen their knowledge of why rounding is necessary and useful. Explain to students the usefulness of approximating numbers rather than giving their exact values. Show the example of attendance at a sporting event in the Student Book. You could collect some more examples of reporting on crowd numbers to emphasize that an approximate value is usually used. To help students understand why approximations are easy to work with, you could give them a list of three large numbers, such as 36214, 19991, and 65021, and ask them to remember them and write them down. Not many students will be able to do so. Next give them the approximations of these numbers (36000, 20000, and 65000) and ask them to remember and write these down. Many more students will be successful, because there is less for them to remember.

Show a number line and display some numbers on it to discuss which ten, hundred, thousand, or ten thousand (and so on) they are nearest to. Talk about the 'halfway mark' for tens, then for hundreds, and so on. Note that the halfway mark always contains a 5 in the place immediately after the level of accuracy we are looking at (so, when looking for the nearest thousand, the halfway mark contains a 5 in the hundreds column). There is no particular reason why a number on the halfway mark must be rounded up, and students may question why this is the case. Explain that this is the convention.

Students may not be familiar with the 'approximately equal to' symbol, \approx . Take a moment to talk about its meaning and give students the opportunity to practise writing it. Look out for students writing an equals sign, such as 36214 = 36000. This is incorrect and the approximately equal to symbol should be used. Emphasize to students that they should always write the degree of accuracy in brackets after their answer, such as $36214 \approx 36000$ (to the nearest ten thousand). Check for and insist on this as a way of teaching clarity of mathematical communication alongside the mathematics itself.

You could give students an example of rounding to the nearest ten, such as a number between 530 and 540. If students need more support, place the number on a 530–540 number line. Mark the halfway point (535) to help students to identify which way to round, and which place column to look at to decide. You could ask students working towards extending outcomes to identify the greatest and least numbers (the lower and upper bounds) that would round to 540.

Example-problem pair

This question involves rounding to the nearest 10000. It focuses on knowing which column to look at to make the decision. If students need more support, suggest that they use a number line. Ask students to try 'Your turn!'.

Your turn! answer

5460000

Reinforce that to find the lower bound and upper bound for a rounded number, students need to identify the specified power of 10 above and below the number. They then find the halfway points between these powers of 10. For example, when a number is 8000 to the nearest 1000, the thousands before and after are 7000 and 9000 and the halfway points are 7500 and 8500. Remind students that the halfway point below will round up to 8000 but the halfway point above will round up to 9000.

After students complete the Fluency questions, use these to check students' understanding before moving on.

1.1.1 Fluency questions: answers

p.5

b 5300

c 460000

d 3000000

2 a 100

b 90

c 6560

3 a 1000

b 83600

c 904675500

4 a 5000

b 13000

c 134915000

5 a 790000

b 9600000

c 23000000

6 3550

7 45500

8 5995

1.1.2 Rounding numbers to decimal places

p.6

Objective

Students will learn to:

round numbers to any number of decimal places.

Rounding to a given number of decimal places uses exactly the same skills as rounding to the nearest 10, 100, 1000, and so on. In fact, rounding to one decimal place could be called rounding to the nearest tenth. It is important that students understand that they must still identify the place of the digit in question and look to the next column to decide how to round.

Students have worked less with decimals than with integers, so they sometimes find it more difficult to round to decimal places. If so, using number lines helps, as does demonstrating the rounding process in a consistent way. Be aware that, to some students, working with decimals can feel very different because of the way the numbers look. If, for example, we round 245 to the nearest 100, we must use zeros as placeholders (200). In contrast, if we round 0.245 to one decimal place, we do not need any placeholders (0.2). Although the process is the same, the answer is visually different and this can cause some confusion. Compare and contrast more integer rounding with decimal rounding if necessary.

Use the example of rounding 0.8075 to different decimal places to show how the number itself does not change, but where we look on the number line does change. Place 0.8075 on an online zoomable number line and show it rounded to one, two, and three decimal places by zooming in. Explain to students that we are gradually making more accurate approximations as we use more decimal places. Point out that this is demonstrated by the fact that the distance from the number to its approximation is getting smaller.

The number line from 1.7568 to 1.7569 in the Student Book helps students to work with more decimal places than they are normally used to. A zoomable online number line can be used to create more similar examples.

Example-problem pair 1

This question considers a decimal between 0 and 1 but uses the same thinking process for deciding which column to look at to round up or down. If students need more practice of rounding, you could ask them to round the same number to other decimal places. Ask students to try 'Your turn!'.

Your turn! 1 answer

0.0236

When rounding to the nearest integer, remind students that they no longer need to see a decimal point because they are deliberately trying to move to a 'whole' number.

Example-problem pair 2

This question focuses on rounding decimals to the nearest integer. Emphasize that the number of decimal places is not important. All that students need to consider is the digit in the first decimal place. Ask students to try 'Your turn!'.

Your turn! 2 answer

340

You could ask students working at extending outcomes to give the greatest and least values that would round to each answer in the Example-problem pairs.

Discuss with students the importance of considering context when rounding. For example, when buying ingredients for a recipe, it is usually better to round up to make sure that you buy enough. However, when considering how many episodes of a show you can watch in a given timeframe, you need to round down to ensure that you do not have to stop midepisode. When rounding in these contexts, it may not be sensible to use a degree of accuracy more precise than integers. For example, it is not possible to reserve 4.5 coaches for a school trip, or buy 3.2 boxes of eggs in a supermarket.

After students complete the Fluency questions, use these to check students' understanding before moving on.

1.1.2 Fluency questions: answers

p.7

1 a 6.6

b 8.91

c 12.0763

2 a 7.2

b 9.25

c 82.165

d 13.0

e 143.646

f 0.10000

3 0.7855

4 3.18749; 3.187549

Support students to use the Reflect, Expect, Check, Explain model by using prompts: Have you seen a question like this before? What has changed from the previous question part? How do you think that will affect the answer? Can you explain why you got that answer?

1			To nearest 10	To nearest 100	To nearest 1000
	а	853902	853900	853900	854000
	b	85390	85390	85400	85000
	C	8539	8540	8500	9000
	d	853	850	900	1000
	е	85	90	100	0
	f	8	10	0	0

		To 1 decimal place	To 2 decimal places	To 3 decimal places
а	2.1032	2.1	2.10	2.103
b	2.2143	2.2	2.21	2.214
C	2.3254	2.3	2.33	2.325
d	2.4365	2.4	2.44	2.437
e	2.5476	2.5	2.55	2.548
f	2.6587	2.7	2.66	2.659
g	2.7698	2.8	2.77	2.770

3	а	1
	b	1
	c	2
	d	2
	е	16
	f	0
4	а	1

b 1.0

c 1.00

d 1.000 **e** 1.0000

f 1.00000 **g** 1.000000

a 3521.5

b 3522

c 3520

d 3500

e 4000

6 a 1500000

b 1950000

c 1995000

d 1999950

e 1999999.5

7 a 3.8

b 4

c 4

Question 1 uses the same digits but removes one digit each time, and asks students to round each number to the nearest ten, hundred, and thousand. Students must focus on the rounding effect of different digits and also the relative size of each number.

Question 2 uses the same pattern in the digits after the decimal point, increasing each digit by one for every part. Students have to think carefully about which digits cause rounding in different directions (up or down).

Question 3 starts with numbers between 1 and 2, requiring students to decide whether to round up or down. It then changes the position of the decimal point.

Question 4 looks at when rounding will change digits other than the digit mentioned in the degree of accuracy. It also emphasizes when trailing zeros must be included in an answer to demonstrate accuracy.

Question 5 asks students to round the same number to different degrees of accuracy. Ensure that students are identifying the digit in the required place each time, and then looking at the digit one place to the right to decide how to round.

2

In **Question 6**, students must identify the lower bound that could round to 2000 000 for different degrees of accuracy. Encourage students to reflect on which digits they should focus on each time.

Question 7 requires students to round the same number in different contexts.

1.1 Which method?: answers

p.9

- **1** 20°C
- **2** 46.1 cm²
- **3 a** Any five numbers in the interval 1.2 < x < 1.25, e.g. 1.21, 1.22, 1.23, 1.24, and 1.201
 - **b** Any five numbers in the interval 1.15 ≤ x < 1.2, e.g. 1.15, 1.16, 1.17, 1.18, and 1.19
- **4 a** \$20.83
 - **b** \$0.51

- 5 3 hours
- 6 a 68 000 000 or 68 million
 - **b** 68 000 000 or 68 million
- **7** 13
- Any three numbers in the interval
 7500 ≤ x < 8500, e.g. 7900, 8010, and 8499.
 - **b** 7500

Question 1 requires students to read a scale on a thermometer. They need to make a rounding decision based on visual placement without knowing the value of the number to be rounded. This reinforces the idea of rounding being based on 'nearest to' on a number line.

Question 2 makes use of rounding in the context of area from Student Book 7, Chapter 6. Explain to students that an answer to one decimal place is appropriate when both lengths are given to one decimal place.

Question 3 asks students to work backwards by creating their own numbers that round to a given value. By asking for numbers greater than and less than 1.2, students have to think about what causes rounding up and what causes rounding down.

Question 4 is a real-world problem that requires some problem solving in part **b**. Students must round and then decide which calculations to do in order to answer the question. Draw students' attention to the key words, such as 'overpay'.

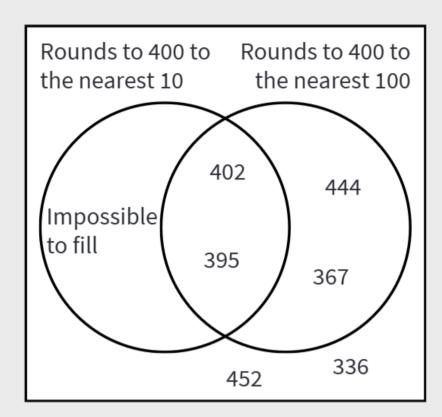
Question 5 links rounding with time. Some students may round down, since they see the digit '4'. If so, either remind them what proportion of an hour 40 minutes is or use a clock face to emphasise that 40 minutes is over half an hour.

It is best to start **Question 6** by writing the number in digits first. This question links with work on place value from Student Book 7, Chapter 1.

Question 7 presents students with a context where it is only possible to buy integer lengths. Students should realize that they need to round up. They are also required to convert between centimetres and metres.

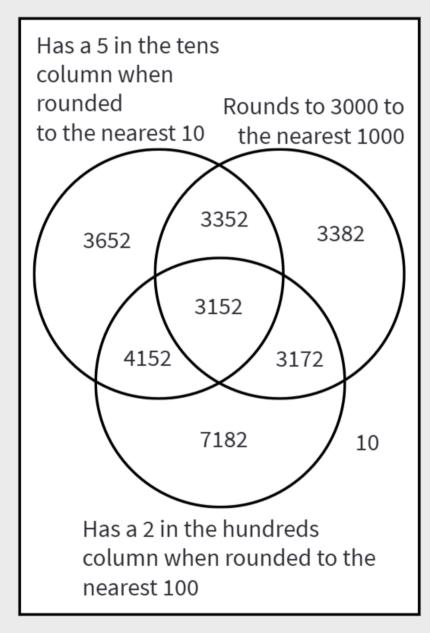
Question 8 is an introduction to the idea of bounds. If students are struggling with part **b**, ask them to test numbers first, then keep going down until they find a number that rounds to 7000. Ask: What do all the numbers that round to 8000 have in common? (The thousands/hundreds digits are between 75 and 84.) Then, for contrast, ask: What do all the numbers that round to 7000 have in common? (The thousands/hundreds digits are between 65 and 74.)

1 There are many answers. Here is one.



The left section is impossible to fill, because any number that rounds to 400 to the nearest 10 will also round to 400 to the nearest 100.

2 There are many answers. Here is one.



- Ava has rounded the 48 in the fraction $\frac{48}{100}$ to the nearest 10 to get $\frac{50}{100}$, which is half, so this is fair to say.
- **4** Derek rounded both numbers to the nearest integer and got 12 g.
- **5 a** E.g. 896
 - **b** E.g. 9997
 - **c** E.g. 892
- **6 a** 1260
 - **b** 1261
 - **D** 1201
 - **c** 1260
 - **d** 1260
 - **e** 1500
 - **f** 0
- **7 a** -3
 - **b** -11
 - c 8
 - **d** 0
 - _ 1
- 8 Student answers

The Venn diagram in **Question 1** encourages students to think about which numbers will round to 400 to the nearest 10 and 100. If students have placed any numbers in the left section, ask: *How would you round that number to the nearest 100?* Students should notice that it will round to 400 and therefore belongs in the intersection.

A way into **Question 2** is to start with the top left section. Students may give a number between 45 and 54, so ask them whether this number belongs in any of the intersections. This will encourage them to consider the other sections in more detail. Ask students what a number would look like that went in the intersection of the top left section and one of the other sections, before considering numbers outside the intersections.

Question 3 should encourage some discussion about the word 'around' and when it is appropriate. Ask: What are the lowest and highest values you would accept as 'around half'?

Question 4 asks students to identify how two numbers have been rounded. Ask them whether or not they think Derek's statement is accurate, or even fair. Students may have different opinions based on their perspective of the scenario. The difference in the amount of fruit is 0.33 g. Ask: *Do you think this is a significant amount? Would your answer change if the values were 11.5 g and 12.49 g?*

There are many different answers for all parts of **Question 5**. To get started, students can write down any number and look at what it rounds to, to the nearest 10 and 100. They can then decide how to change their number to fit the criteria.

Question 6 introduces some more unusual types of rounding. A discussion about the halfway point is helpful here. For example, ask: When rounding to the nearest even number, what would round up and what would round down? Placing numbers on the number line and marking intervals such as even numbers, or multiples of five for part **c**, can be helpful here. Part **f** reminds students that 0 is a multiple of every number and is therefore an acceptable answer to rounding questions.

Students could use a number line for **Question 7**. Ask students to label the negative numbers and then reflect them in a line of symmetry going through 0. You could then ask: What would happen to the corresponding positive number?

Question 8 asks students to write their own word problems where the context dictates that the answer must be rounded down. Listen for them suggesting contexts where only integer answers make sense and there are constraints such as time or money. For example, how many of an item can be bought with a given amount, or how many times a task can be repeated in a given time. Ask students to swap their questions with a partner and answer each other's questions.

Rounding to significant figures

Students take their knowledge of rounding techniques and apply it to the type of rounding they will employ the most often throughout this course and beyond: significant figures. They will be introduced to the concept of significant figures and how rounding to significant figures encompasses all the types of rounding they have used so far. They will then learn to round integers and decimals to a given number of significant figures.

	Learning outcomes			
Learning objectives	Developing	Secure	Extending	
Understand significant figures	Identify the first significant figure in any given integer or decimal	Identify the significant figures in integers and decimals	Explain why some zeros are significant and other zeros are not significant	
	E.g. Circle the first significant figure in each of these numbers:	E.g. How many significant figures are there in each of these numbers?		
	100 000.3, 608, 0.1001, 0.000 085 02	100 000.3, 608, 0.1001, 0.000 085 02		
Round integers using significant figures	Round integers to a given number of significant figures that do not contain the digits 0 or 9 E.g. Round 3487 to one significant figure.	Round any integer to a given number of significant figures, including where some digits are 9 or 0 E.g. Round 3099 to two significant figures and three significant figures.	Reason whether a given number could have been rounded to a certain number of significant figures E.g. A number has been rounded to give the result 840. How many significant figures were used?	

Tier 2 vocabulary	Tier 3 vocabulary
	non-zero, placeholder, significant figure

Classroom resources

Equipment

You will need:

calculators

Models

• number lines

Digital resources



My self-study quiz, Example-problem pairs, Exercise handout, Extra fluency questions, videos

Objective

Students will:

• understand significant figures.

Before rounding to a given number of significant figures, students must first understand what a significant figure is. The simplest explanation is that the first significant figure is the first non-zero digit, although this can bring some confusion later where some students think that only non-zeros are significant. Emphasize that all digits can be significant, including zeros, but that significance only begins once we reach the first non-zero digit.

Discuss the value of the first significant digit in the examples given so that students see that its value changes. This prepares students for the fact that a statement such as 'round to one significant figure' can mean the same as 'round to the nearest 1000', or 'round to three decimal places', depending on the value of the first significant figure. This is where the power of significant figure rounding lies.

Ask students to write three numbers whose first significant digit is a 3. Ask: Can you make the numbers decimals? Can you make the numbers all between 0 and 1000? Can their first significant digits all have the same value? A different value?

In Student Book 7, Chapter 1, students learned about the idea of placeholder zeros, which keep the other digits in the correct place value columns. These placeholders may appear at the start of a decimal less than 1, or in between non-zero digits. Show students examples such as 203 and 0.004, emphasizing how the value changes if the zeros are removed.

Placeholders are easiest to contrast with significant figures using decimals between 0 and 1, because all the zeros before the first significant figure are placeholders. It is a misconception that all zeros are placeholders in rounded integers. For example, if we round 1990 to one significant figure, we get 2000 and all three zeros are placeholders. If, however, we round 1990 to two significant figures, we still get 2000, but this time the first two digits are significant and only the last two zeros are placeholders. For this reason, it is best to avoid a discussion of placeholders and significant figures in integers until we have started to round in the next subsection.

Example-problem pair 1

This question checks that students can identify the first significant figure by applying the 'first non-zero digit' rule. The 'Your turn!' gives them further practice.

Your turn! 1 answers

- **a** 2, worth 200000
- **b** 1, worth 0.0001

Make sure students realize that once significant numbers have started, all digits that follow are significant, including any zeros.