Oxford International Resources

7

# Maths

## Teacher's Guide

Lower Secondary

OXFORD

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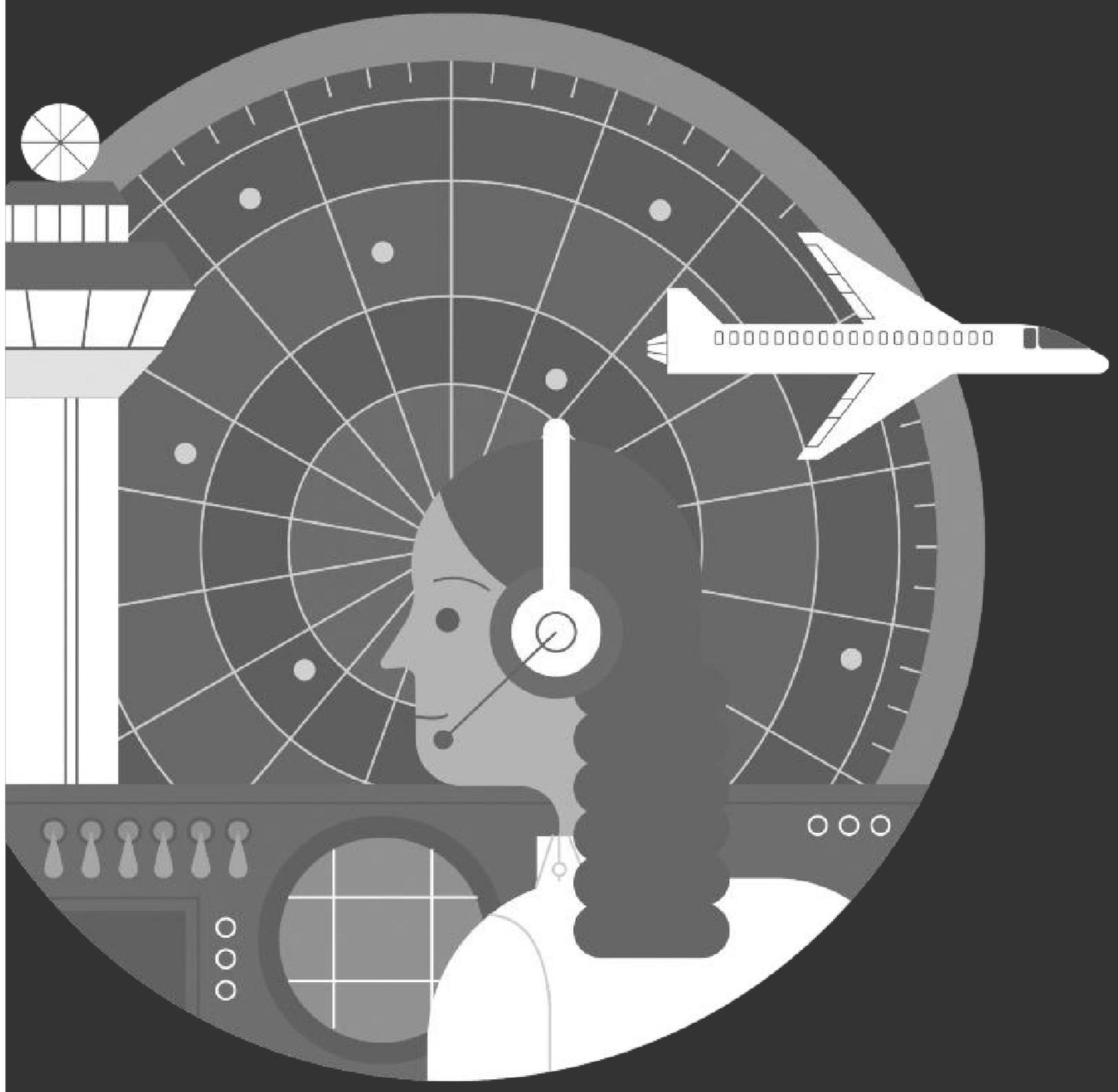


Craig Barton

Jemma Sherwood
Dan Draper
Charlotte Hawthorne
Helen Konstantine
Katie Wood



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# Contents

# Introduction

## The joy of learning maths

We are living in an ever-changing world, where the way we work, live, learn, communicate, and relate to one another is constantly shifting. In this climate, we need to instil in our learners the skills to equip them for every eventuality so they are able to overcome challenges, adapt to change, and have the best chance of success. To do this, we need to evolve beyond traditional teaching approaches and foster an environment where students can start to build lifelong learning skills. Students need to learn how to learn, problem-solve, be agile, and work flexibly. Going hand in hand with this is the development of self-awareness and mindfulness through the promotion of wellbeing to ensure that students learn the socio-emotional skills to succeed.

## Teaching and learning with *Oxford International Maths*

This course is suitable for use alongside the Oxford International Curriculum and the English National Curriculum. The books for each year (or stage) follow the scheme and meet the learning objectives for both curricula. Objectives are written in student-friendly language in the Student Book.

The Lower Secondary course is designed for students aged 11 to 14. Each year has a Student Book and a Teacher's Guide. There are also numerous digital resources and sources of support on www.kerboodle.com. These three core elements provide a cohesive offer that supports learners to develop and consolidate knowledge and skills, and to draw connections between topics.

*Oxford International Maths at Lower Secondary* is a mastery course. The term 'mastery' has been drawn from teaching approaches in countries where maths performance is high. The essence of mastery is to produce students who have deep conceptual understanding and procedural fluency through learning in a collaborative and problem-solving context. Mastery learning incorporates exposure to different methods of solving a problem, mathematical dialogue, and explanation.

This course has six main aims:

### 1 Provide clear and coherent curriculum pathways

Close attention is paid to the sequencing of concepts, connections with prior learning, common difficulties and misconceptions, and consistent use of key models and language.

Pre-requisite, current, and future learning are signposted to ensure that learners are clear where they are on their learning journey. Checks and diagnostic questions help teachers identify where learners are before teaching a new topic, and integrated assessment provides regular feedback on learners' progress against the curriculum pathway.

### 2 Hold high expectations, aspirations, and ambitions for all learners

Developing, secure, and extending (DSE) learning outcomes underpin each learning objective to ensure that the same curriculum is accessible to all, with the aim that all learners achieve a secure level.

Support for reactivating prior learning and identifying misconceptions at the start of each topic helps reduce the attainment gap and ensures that all learners are at a suitable starting point for the new content. Carefully graduated practice allows all learners to build confidence while also providing stretch for those who are ready for more challenge. Differentiation is supported through questions targeted at each of the DSE outcomes.

### 3 Support engaged, self-regulated, and metacognitive learning

The Reflect, Expect, Check, Explain (RCE) framework has been developed by Craig Barton. It encourages metacognitive thinking where students think about their learning, feel empowered to reflect on their strengths and areas for development, and make connections across topics.

### 4 Promote development of learner identity and identification with each subject

The relevance of maths to learners is highlighted throughout curriculum components and resources, including student-facing facts and visuals that encourage thinking about real-world applications and provide prompts for discussion.

### 5 Enable responsive teaching and learning that continually evolves and improves

Resources are provided to help teachers check understanding at every stage in the learning process, and guidance about common difficulties and misconceptions supports teachers to respond to learners' needs.

Regular formative assessment on your Kerboodle subscription provides learners with personalized next steps to secure or extend their learning. Detailed reporting on Kerboodle supports both teachers and learners.

### 6 Inspire fascination and awe and wonder in the world around us

This course aims to stimulate fascination through a strong focus on thinking and working mathematically. Opportunities to develop the skills associated with working like a mathematician are embedded throughout.

## Teaching techniques

### Grouping students to promote a growth mindset

It is expected that you will use a variety of student groupings. It is important that you are active in deciding which form of grouping is appropriate, depending on the activity. In this way, students will learn how to work in a variety of ways and with a range of different students.

There are three main ways of grouping students:

- **Friendship groups** are most appropriate for activities in which students have been given some element of choice, for example, if they are carrying out some research for a data handling project. This grouping is the default if teachers do not actively group students.
- **Ability groups**, or groups based on students' prior experience, may be helpful if the lesson requires a very specific prior knowledge. You can group together the students that you know have this knowledge, and they can then work with minimal guidance from you. This will allow you to focus on groups who need additional support.
- **Mixed-attainment groups** are encouraged for the majority of activities. This form of grouping is also favoured by those following a mastery approach. Working in collaborative, all-attainment groups also supports students' wellbeing and promotes a growth mindset, as described in research by Carol Dweck. She found that students who were grouped by ability tended to stay in those groupings throughout their school life, and regard themselves as having a fixed ability that could not be changed. This has dire consequences for students in middle or lower sets. When placed in mixed-ability groups, all students can develop a growth mindset that enables them to believe they can learn and improve, whatever their starting point (Dweck, C., 2007. 'The Perils and Promise of Praise'. *Educational Leadership.* October 2007, 65(2), 34–39). A growth mindset is promoted when students do not feel that their future success is predicated on prior achievement. This kind of grouping is particularly helpful for students new to English, as those who are less confident speaking the language will be able to hear their more confident peers using mathematical vocabulary. Research has shown that mixed-attainment groups benefit both high attainers, who become more secure in their maths knowledge through explaining their thinking to peers, and those less secure in their maths knowledge, through peer teaching.

### Asking effective questions

The most skilled maths teachers can ask open questions to elicit students' current understanding. Skilful open questioning also allows students to articulate their current understanding carefully, and through this process either consolidate their understanding or come to realize where they have made a mistake. The following list offers a series of open questions that can be used whatever maths you are teaching.

- *How are these the same/different?*
- *What would happen if …?*
- *How else could you have done that?*
- *Why did you …?*
- *How did you …?*
- *How do you know that is correct?*

If you want students to check their solutions and consolidate their learning, ask them to explain how they reached their solution to a friend. Similarly, to support students in reflecting on their learning, you might ask the following:

- *What maths did you use to solve the problem?*
- *What new maths did you learn?*
- *What key words did you use?*
- *What was the most challenging part of the activity?*
- *What did you do when you got stuck?*
- *What other questions could you ask?*
- *Did this remind you of any other areas of maths?*

## Differentiation

Differentiation is closely linked to inclusion: ensuring all students have access to the curriculum. This means that learning and teaching approaches must consider individual needs. Not all students will learn at the same pace or in the same ways.

This course supports the following approaches:

- **Differentiation by task** Content can be adjusted for some students to provide sufficient support or adequate challenge. The 'Fluency questions' in the Student Book are ramped, starting with questions aimed at less able students and finishing with 'stretch zone' questions. The latter are designed to extend more confident students and challenge them to think more deeply. 'Which method?' questions are also aimed at more confident students and require prior knowledge. 'Expert practice' questions are designed to be accessible to all, but self-differentiating depending on approach. For less able students, prioritize the questions in the Example-problem pair (EPP) grids.
- **Differentiation by outcome** This allows all students to tackle the same tasks, but with differentiated learning outcomes. There are three bands of differentiation for each learning objective: developing, secure, or extending. The differentiated outcomes are provided for each lesson in this Teacher's Guide. 'Secure' indicates that students have a secure grasp of the knowledge or skills specified. The band working towards 'secure' is 'developing', and the band moving past 'secure' is 'extending'.
- **Differentiation by support** This means providing more or less support as students are carrying out a task. Advice on this is provided for each lesson in this Teacher's Guide. For additional practice, support sheets are available on Kerboodle to give less able students further opportunities to reach a secure understanding of new or challenging concepts in their own time. These worksheets can be tackled independently or used in adult-led, small-group sessions.

## Assessment

Assessment is an essential part of learning. Without being able to check progress, teachers and students will not be able to identify areas of strength and areas in need of development.

Each activity – group and individual – can be assessed through observation, questioning, and progress notes. Written or drawn responses for each activity can be assessed/marked using the school's marking policy; and chapter, end-of-term, and end-of-year judgements made about individual and class progress.

Feedback is a crucial aspect of assessment. This should be as positive and encouraging as possible, in which clear targets are identified. Involve students in assessment and target setting – assessment is done *with* learners, not done *to* learners.

### Formative assessment

This takes place during learning and is used to address issues as they arise. This means learning and teaching can be modified during lessons to better meet students' needs. Feedback is ongoing.

Each activity within the Student Book provides opportunities for formative assessment and

| | Learning outcomes | | |
|---|---|---|---|
| **Learning objective** | **Developing** | **Secure** | **Extending** |
| Learners at this stage… | …are working towards secure knowledge and understanding but need more support to achieve this. | …have a secure knowledge and understanding. | …are working beyond expectations, and their knowledge and understanding can be stretched and challenged. |
| *e.g. Understand and list multiples of a number* | *List the multiples of a given number inside the times tables and use the term 'multiple' correctly* | *Understand multiples can be outside times tables up to 12* | *Understand that two multiples of a number add to give another multiple of the number* |

feedback. You can do this by listening to explanations or paired discussions; observing students' workings; and assessing outcomes. Individual questions can be used to monitor understanding and identify misconceptions. These can be addressed as they are noted.

### Summative assessment

This is used to measure or evaluate student progress at the end of a process – for example, when a chapter is completed or at the end of a year. Summative assessment compares students' attainment against a standard or benchmark.

The 'What have I learned?' pages at the end of each chapter can be used for summative assessment. You can record which questions each student is answering correctly and use this to measure individual attainment. It can also indicate how well the class is progressing through the work. In this way, the assessment can inform individual interventions (extra support for a student) or whole-class interventions (reviewing work that is not well understood).

## How to support non-native English speakers

Ministries of Education at both local and national level are increasingly adopting the policy of English Medium Instruction (EMI), either for one or two subjects or across the whole curriculum.

In international schools, it is likely that students do not share a mother tongue with each other or perhaps the teacher. English is chosen as the medium of instruction to level the playing field and provide the opportunity to develop proficiency in an international language.

This does not mean that the maths teacher is expected to replace the English teacher, or to have the same skills or knowledge of English. However, they do need to become more language aware. This raises significant challenges, including:

- the teacher's knowledge of English
- students' level of English (which may vary considerably in international schools)
- resources that provide appropriate language support
- assessment tools that ensure that it is the content and not the language being tested
- differentiation that acknowledges different levels of proficiency in both language and content.

### Language in the classroom

Using English in the classroom is very important as it provides exposure to an additional language (often a student's second or third), which plays a valuable role in language acquisition. The 'teacher talk' for purposes such as checking attendance and collecting homework does not have to be totally accurate or accessible to students. However, when teaching mathematical concepts, it is essential that the 'teacher talk' is comprehensible. The following strategies can help:

- simplify your language
- use short, simple sentences and project your voice
- paraphrase as necessary
- use visuals, the board, gestures, and body language to clarify meaning
- repeat as necessary
- plan before the lesson
- prepare clear instructions and check understanding.

### Creating a language-rich environment

Providing a colourful and visually stimulating environment for students becomes even more important in the EMI classroom. Posters, lists of key words and structures, displays of students' work, and signs and notices in English all maximize students' exposure to English and, in big or small ways, contribute to their language acquisition.

### Planning

In your planning, identify each language demand (LD). You will need to think about what language students will need to understand or produce, and decide how best to scaffold the learning to ensure that language does not become an obstacle to understanding the concept. This kind of language support (LS) goes beyond the familiar strategy of identifying key vocabulary.

***Support for listening and reading***

Listening and reading are receptive skills, requiring understanding rather than production of language.

If students need to listen to or read in English, ask yourself the following questions:

1. Do I need to teach any vocabulary before they listen/read?
2. How can I prepare them for the content of the text so that they are not listening 'cold'?

3 Can I provide visual support to help them understand the key content?

4 How many times should I ask them to listen/read?

5 What simple question can I set before they listen/read for the first time to focus their attention?

6 How can I check more detailed understanding of the text? Can I use a graphic organizer (e.g. tables, charts or diagrams) or gap-fill task?

7 Do I need to differentiate the task for developing and extending students?

8 Can I make the tasks interactive through groupwork or games?

9 How can I check their answers and give feedback?

***Support for speaking and writing***

Speaking and writing are productive skills and may need more language input from the teacher. You will need to think in detail about what language the task requires (language demands, LD) and what strategies you will use to help students use English to perform the task (language support, LS).

Ask yourself the following questions:

1 What vocabulary does the task require? (LD)

2 Do I need to teach this first? How? (LS)

3 What phrases/sentences will they need? Think about the language for learning maths (e.g. predicting and comparing). What structures do they need for these language functions? (LD)

4 While I am monitoring this task, is there any way I can provide further support to less confident students? (LS)

5 What language will students need to use at the feedback stage (e.g. when they present their task)? Do I need to scaffold this? (LD, LS)

## Teaching vocabulary and structures

***Vocabulary***

Learning key vocabulary is central to EMI, and 'learning' means more than simply understanding the meaning. Knowing a word also involves being able to pronounce it accurately and use it appropriately. Aim to adopt the following strategies:

- Avoid writing a vocabulary list on the board at the start of a topic and 'explaining' it. The vocabulary should be introduced as and when it arises. This helps students associate the word or phrase with the concept and context.
- Record the vocabulary clearly on the board. Check your pronunciation and spelling.
- Give students a chance to say the word once they have understood it. The most efficient way to do this is through repetition drilling.
- Use visuals whenever possible to reinforce students' understanding of the word or mathematical concept.
- Advise students to record the vocabulary systematically in their glossaries under chapter or topic headings.
- Remember to recycle and revise the vocabulary.

***Structures***

Students will need to use phrases and sentence frames to discuss or write about their learning in maths, including these structures:

*X is the same as Y.*

*The next number in the sequence is ... because ...*

*I predict that X will happen.*

*If X happens, then Y happens.*

*The next step is …*

Build up these banks of common maths phrases and remind students to record them. You do not have to focus on grammar here as the language can be taught as 'chunks' rather than specific grammatical structures.

# Component overview

## Student Books

The Student Books are textbooks for students to read and use. They include everything you need to deliver the course to your students, guide their activities, and assess their progress.

| Student Book | Typical student age range |
|---|---|
| Student Book 7 | Age 11–12 |
| Student Book 8 | Age 12–13 |
| Student Book 9 | Age 13–14 |

## Teacher's Guides

There are three Teacher's Guides, corresponding to the three Student Books. Each Teacher's Guide includes:

- advice on delivering maths lessons effectively for EAL students

- a brief introduction to each chapter, including pre-requisite knowledge and mathematical concepts that will be revisited, an introductory activity using a picture prompt, teaching strategies, common learning misconceptions and real-world applications of maths
- guidance on teaching each Student Book topic, including student learning objectives and outcomes, recommended scaffolding, answer keys and approaches to problem-solving.

## Digital

Kerboodle online learning (www.kerboodle.com) provides engaging digital books, lesson resources, and a comprehensive assessment package.

*Digital books*

- **For the teacher:** You can access the Student Books and Teacher's Guides as digital books. The digital books show the course content on screen, making it easier for you to deliver engaging lessons. A set of tools (e.g. sticky notes, bookmarks, pen features, zoom in, and spotlight text) is available to personalize your digital book and make notes. You can share your notes or hide them from view.
- **For the students:** Students can access the Student Books as digital books for use at home.

*Resources*

- Videos – on each topic, also integrated into students' adaptive learning journey
- Exercise handouts – useful visual aids and additional scaffolding for answering the Student Book questions
- Support worksheets and answer keys – extra fluency questions for students who need more practice at developing-level questions
- Example-problem pairs worksheets – additional practice and support for completing EPP grids
- Vocabulary quizzes – for each chapter, to assess students' understanding of key terms
- Mapping to the English National Curriculum, Cambridge international curriculum, and Oxford International Curriculum
- Guidance on how the series supports progression to further study at iGCSE
- Letters to parents/carers to introduce the course and offer guidance on home learning.

***Assessment and adaptive learning journey***

With a Kerboodle login, you can access all the quizzes and tests. First, you will need to import class registers and create user accounts for your students. Once your classes are set up, you can assign students assessments to complete.

Our assessment model combines formative and summative practices. An additional element is regular, low-stakes quizzing aimed at helping students retain new concepts. The formative assessment comprises:

- My self-study quizzes at the end of each topic ask students questions that are relevant to the learning objectives they have just covered. Students' scores will generate either a 'developing to secure' next-step intervention, or a 'secure to extending' next-step intervention. The teacher will also see a breakdown of how students are performing against each of the learning objectives.
- Formative tests which cover content from the whole chapter. Similarly, students will be assigned a next-step intervention according to their score.

Quizzes and tests are auto-marked. Following either assessment type, students are offered personalized next steps. They can consolidate their knowledge if they are at a developing level, or challenge themselves if they have demonstrated secure knowledge.

At the end of each chapter, there is a paper-based summative assessment designed to evaluate understanding of the whole chapter.

***Reporting and insights***

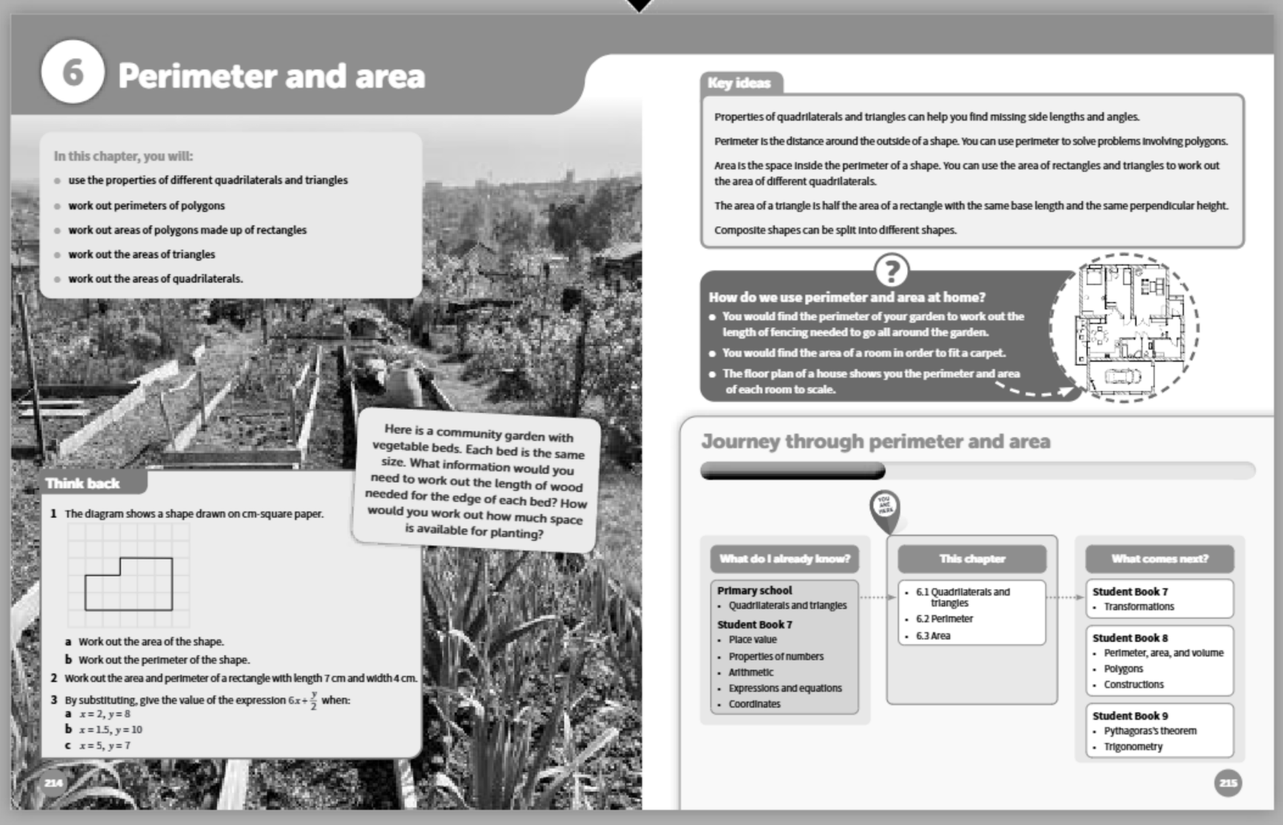
The formative assessment data will feed reporting on Kerboodle and give insights into strengths and areas for development. The data is broken down into learning objectives, and will support you in diagnosing learner needs and focusing your intervention accordingly.

# Tour of a Student Book

**Chapter opener**

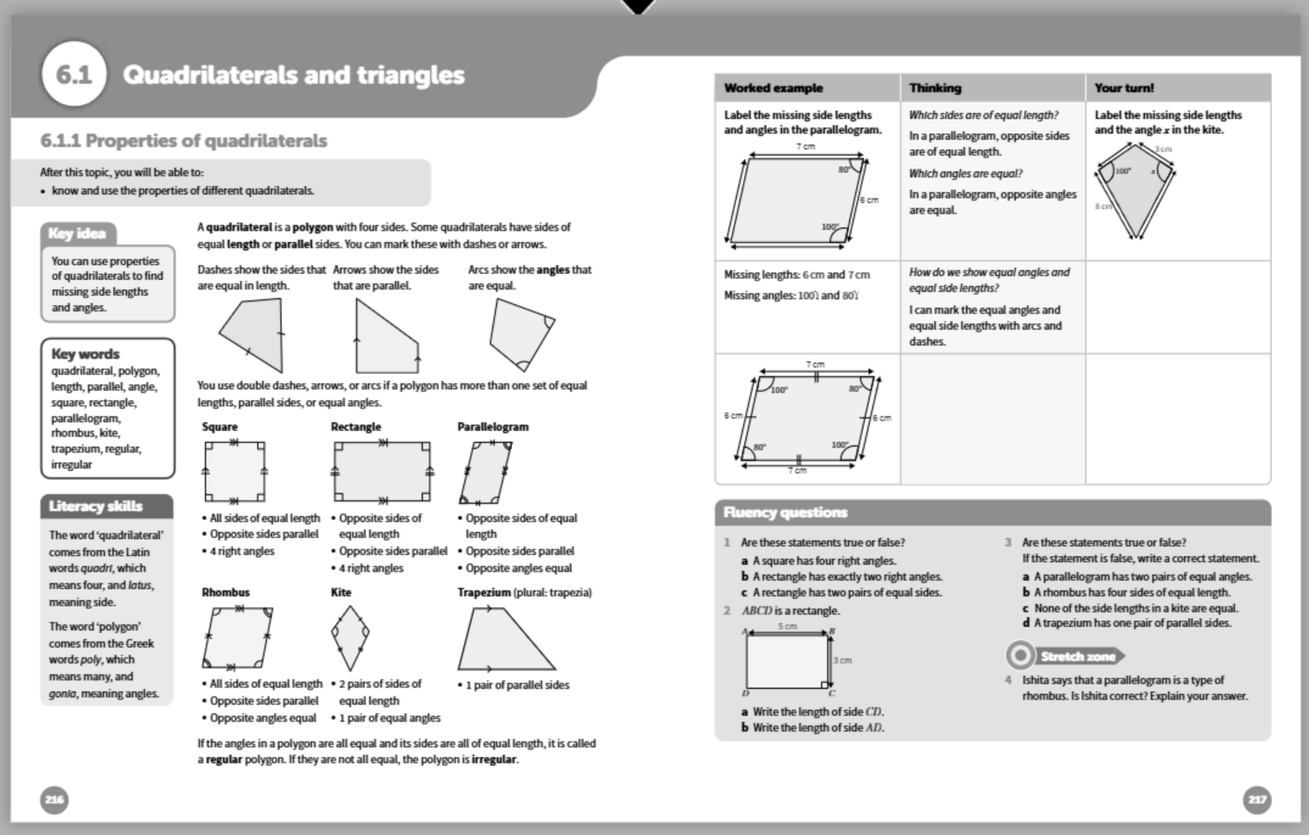
This explains to students what is coming up in the chapter. The 'Learning journey' map shows clearly what maths students should already know from previous learning, the new topics they will study in the chapter, as well as the next steps in their maths learning.

The 'Think back' questions help students recall existing knowledge. This feature will warm up their thinking and alert you to any gaps in their learning before carrying on.




**Lesson pages**

These pages guide students through a particular topic in each chapter. Simplified language is clear and accessible for English language learners, to ensure that a developing understanding of English does not get in the way of grasping key concepts. Skills boxes and fluency questions can then be used to check students' understanding of what they have just read and to stretch their thinking further.




Learning objectives for the lesson are clearly set out at the start and summarized in the Key idea box.

Key words boxes highlight the main maths vocabulary for the lesson. These words are also found in the Student Book glossary.

**Intelligent practice, Which method?, Expert practice**

After the lesson pages for each topic, there are three different types of exercise for students to apply and practise the maths they have just learned:

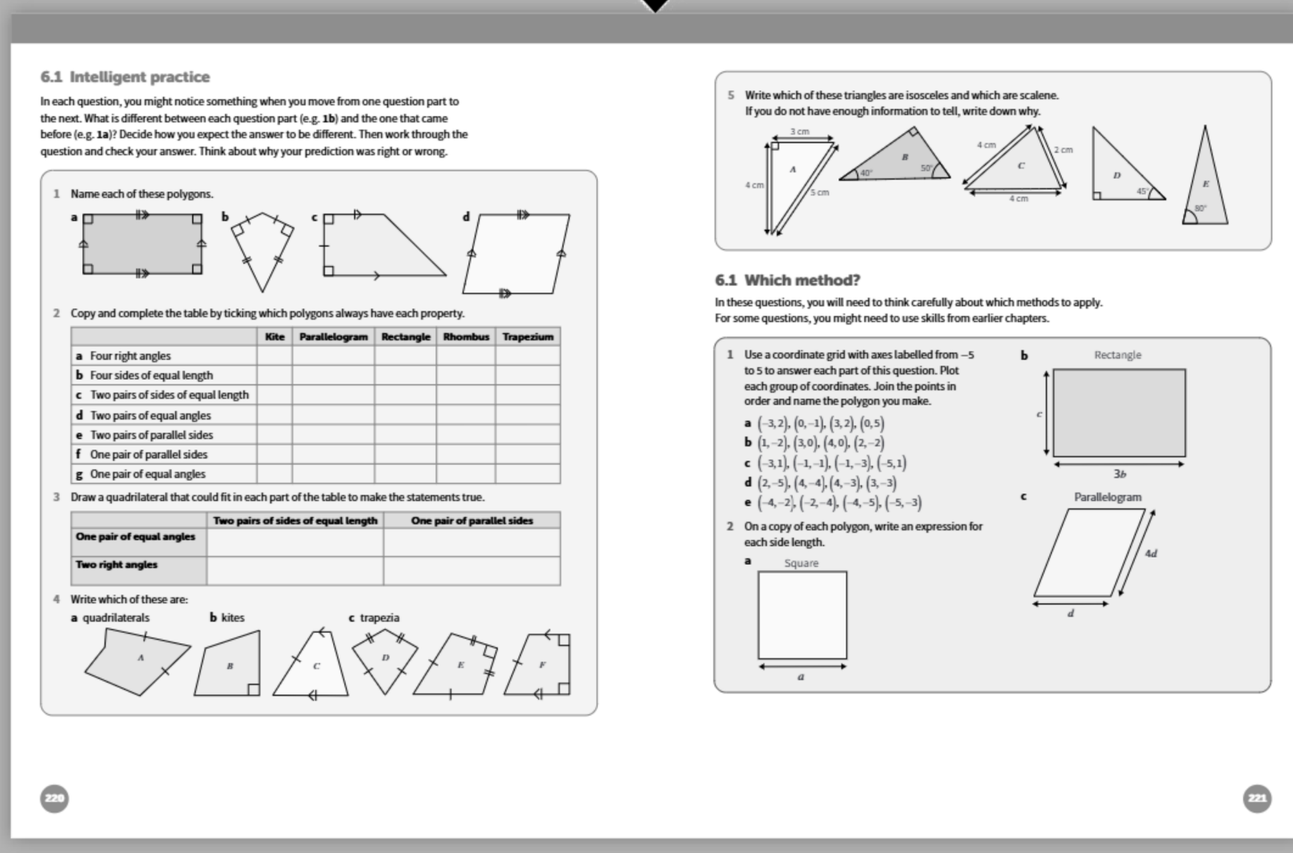
**1** Intelligent practice

**2** Which method?

**3** Expert practice

Each exercise works in a particular way to help the brain make connections, remember the topic, and recognize when to use it.

Find more information about these exercises on p.xiii of this Teacher's Guide.



6.1 Intelligent practice

In each question, you might notice something when you move from one question part to the next. What is different between each question part (e.g. **1b**) and the one that came before (e.g. **1a**)? Decide how you expect the answer to be different. Then work through the question and check your answer. Think about why your prediction was right or wrong.

1 Name each of these polygons.

a b c d

2 Copy and complete the table by ticking which polygons always have each property.

| | Kite | Parallelogram | Rectangle | Rhombus | Trapezium |
|---|---|---|---|---|---|
| a Four right angles | | | | | |
| b Four sides of equal length | | | | | |
| c Two pairs of sides of equal length | | | | | |
| d Two pairs of equal angles | | | | | |
| e Two pairs of parallel sides | | | | | |
| f One pair of parallel sides | | | | | |
| g One pair of equal angles | | | | | |

3 Draw a quadrilateral that could fit in each part of the table to make the statements true.

| | Two pairs of sides of equal length | One pair of parallel sides |
|---|---|---|
| One pair of equal angles | | |
| Two right angles | | |

4 Write which of these are:

a quadrilaterals b kites c trapezia

A B C D E F

5 Write which of these triangles are isosceles and which are scalene. If you do not have enough information to tell, write down why.

A: 3 cm, 4 cm, 5 cm; B: 40°, 50°; C: 4 cm, 2 cm, 4 cm; D: 45°; E: 80°

6.1 Which method?

In these questions, you will need to think carefully about which methods to apply. For some questions, you might need to use skills from earlier chapters.

1 Use a coordinate grid with axes labelled from −5 to 5 to answer each part of this question. Plot each group of coordinates. Join the points in order and name the polygon you make.

a (−3, 2), (0, −1), (3, 2), (0, 5)
b (1, −2), (3, 0), (4, 0), (2, −2)
c (−3, 1), (−1, −1), (−1, −3), (−5, 1)
d (2, −5), (4, −4), (4, −3), (3, −3)
e (−4, −2), (−2, −4), (−4, −5), (−5, −3)

b Rectangle: c, 3b

c Parallelogram: 4d, d

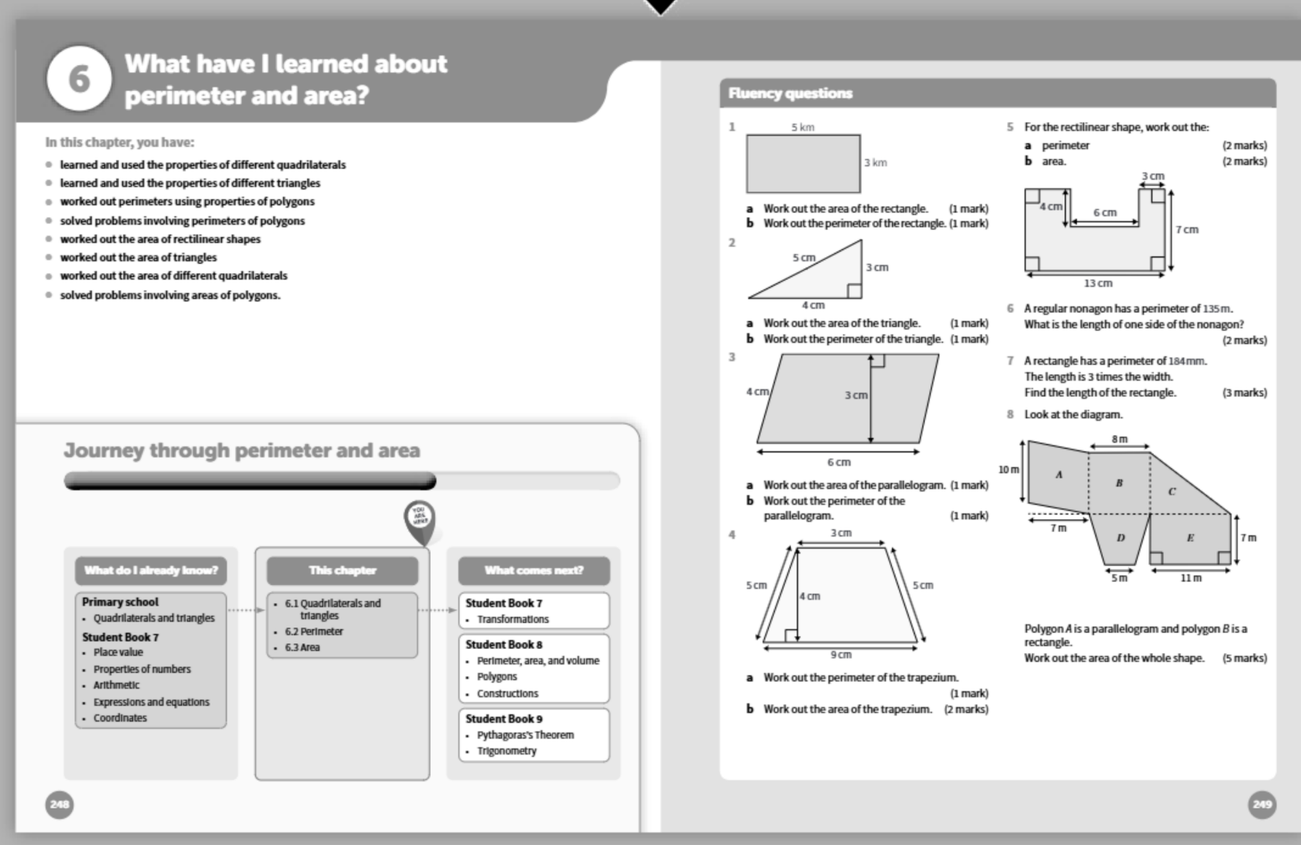
2 On a copy of each polygon, write an expression for each side length.

a Square: a



**What have I learned? pages**

These pages summarize the content that students have learned so far and show how they have progressed in their learning journey. Each chapter concludes with exam-style questions to test how well students have learned and understood the topics, and to keep track of their overall progress.



6 What have I learned about perimeter and area?

In this chapter, you have:

- learned and used the properties of different quadrilaterals
- learned and used the properties of different triangles
- worked out perimeters using properties of polygons
- solved problems involving perimeters of polygons
- worked out the area of rectilinear shapes
- worked out the area of triangles
- worked out the area of different quadrilaterals
- solved problems involving areas of polygons.

Journey through perimeter and area

| What do I already know? | This chapter | What comes next? |
|---|---|---|
| **Primary school**<br>• Quadrilaterals and triangles<br>**Student Book 7**<br>• Place value<br>• Properties of numbers<br>• Arithmetic<br>• Expressions and equations<br>• Coordinates | • 6.1 Quadrilaterals and triangles<br>• 6.2 Perimeter<br>• 6.3 Area | **Student Book 7**<br>• Transformations<br>**Student Book 8**<br>• Perimeter, area, and volume<br>• Polygons<br>• Constructions<br>**Student Book 9**<br>• Pythagoras's Theorem<br>• Trigonometry |

Fluency questions

1 5 km, 3 km
a Work out the area of the rectangle. (1 mark)
b Work out the perimeter of the rectangle. (1 mark)

2 5 cm, 3 cm, 4 cm
a Work out the area of the triangle. (1 mark)
b Work out the perimeter of the triangle. (1 mark)

3 4 cm, 3 cm, 6 cm
a Work out the area of the parallelogram. (1 mark)
b Work out the perimeter of the parallelogram. (1 mark)

4 3 cm, 5 cm, 4 cm, 5 cm, 9 cm
a Work out the perimeter of the trapezium. (1 mark)
b Work out the area of the trapezium. (2 marks)

5 For the rectilinear shape, work out the:
a perimeter (2 marks)
b area. (2 marks)
3 cm, 4 cm, 6 cm, 7 cm, 13 cm

6 A regular nonagon has a perimeter of 135 m. What is the length of one side of the nonagon? (2 marks)

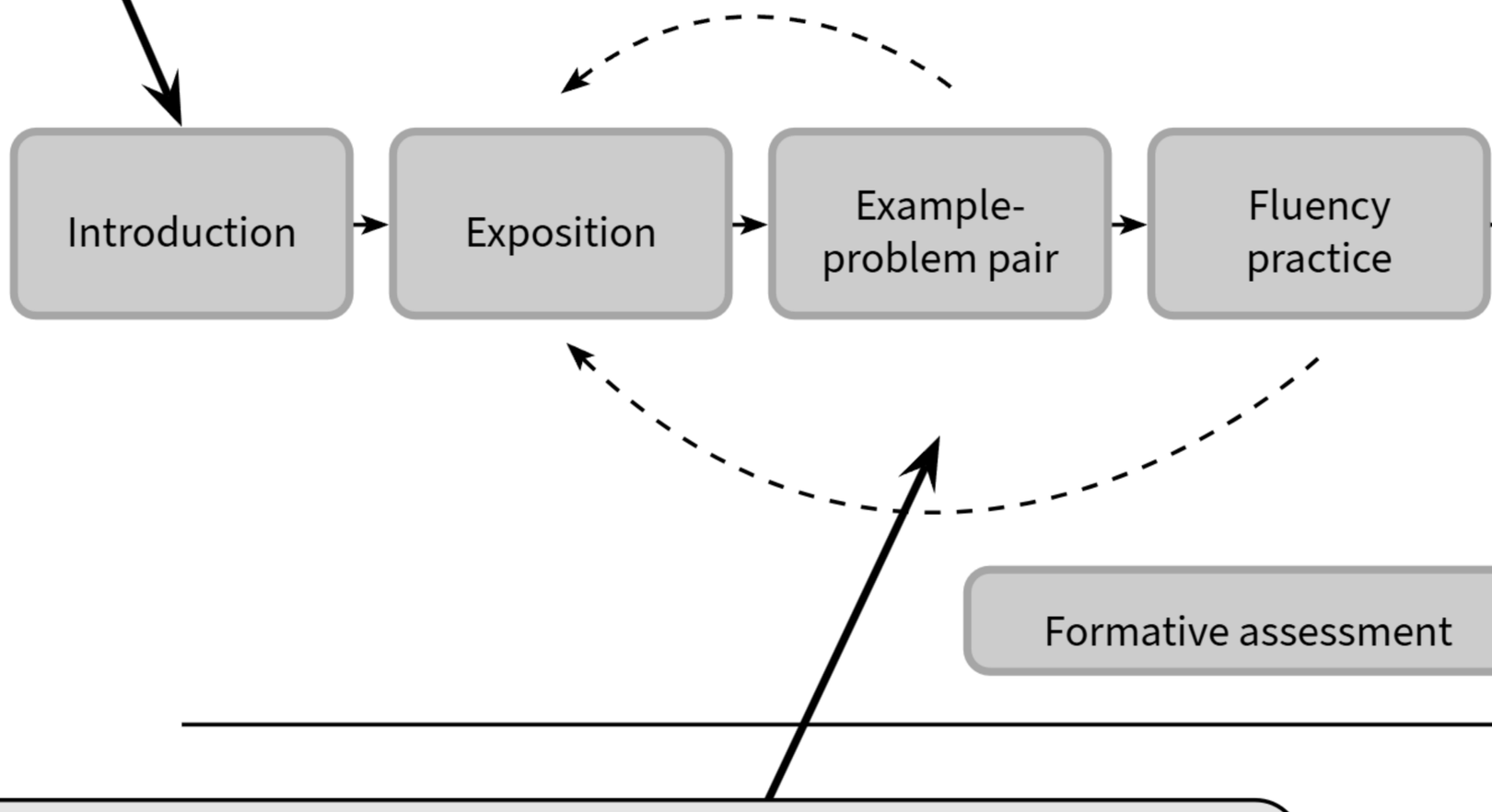
7 A rectangle has a perimeter of 184 mm. The length is 3 times the width. Find the length of the rectangle. (3 marks)

8 Look at the diagram.
8 m, 10 m, 7 m, 5 m, 11 m, 7 m
Polygon *A* is a parallelogram and polygon *B* is a rectangle.
Work out the area of the whole shape. (5 marks)

# The Learning Episode

**Introduction**
*To stimulate interest in a way that helps students feel confident that they can be successful in the maths that will follow.*

- **Link to the big picture**
  The big picture in the chapter opener aims to provide both a purpose to students' study and a platform upon which to build strong memories (e.g. visual examples).
- **Tell a story**
  This will make the information easier to comprehend and remember (e.g. personal reflection – memories of learning it as a child or how you used the knowledge recently).
- **Provide a question hook**
  This should be short and snappy, to spark student interest. You can use the questions linked to the big picture or real-world application in the chapter opener.
- **Discuss the etymology of key words**
  This will make it easier to remember the words' meaning, showing students how words connect to other areas of maths and the world. You can use the Literacy skills boxes throughout the Student Book.




**Example-problem pair**
*To model how to do a new method in a way that allows students to be actively involved and attend critical elements.*

1 **Silent teacher**
  Run through the example without verbalizing any questions, but pausing and turning to students to cue them to think about what has happened.
2 **Narration**
  Go through the example again, this time using thinking prompts to draw students' attention to critical elements of the working out.
3 **Read the maths**
  Students use the thinking prompts to add annotations to the example (downloadable worksheets are available on Kerboodle) so that it makes sense.
4 **Your turn**
  Students complete the question using the model example to support them with answering the question.
5 **Share learning**
  Use a visualizer and examples of student work to highlight examples of good practice, misconceptions, etc.

**Intelligent practice**

*These are sequences of questions that enable students to gain practice in carrying out a mathematical method, while providing opportunities to think mathematically.*

**1 Model the relationship**
Model the *Reflect, Expect, Check, Explain* process first using an example question.

**2 Silent practice**
Students complete questions in silence, allowing them to think before they ask for help and make connections at the points at which they are ready.

**3 Paired discussion**
Students rehearse and modify their explanations, and listen and learn from others.

**4 Discuss relationships**
Reveal the answers and delve deeper into one or two relationships.

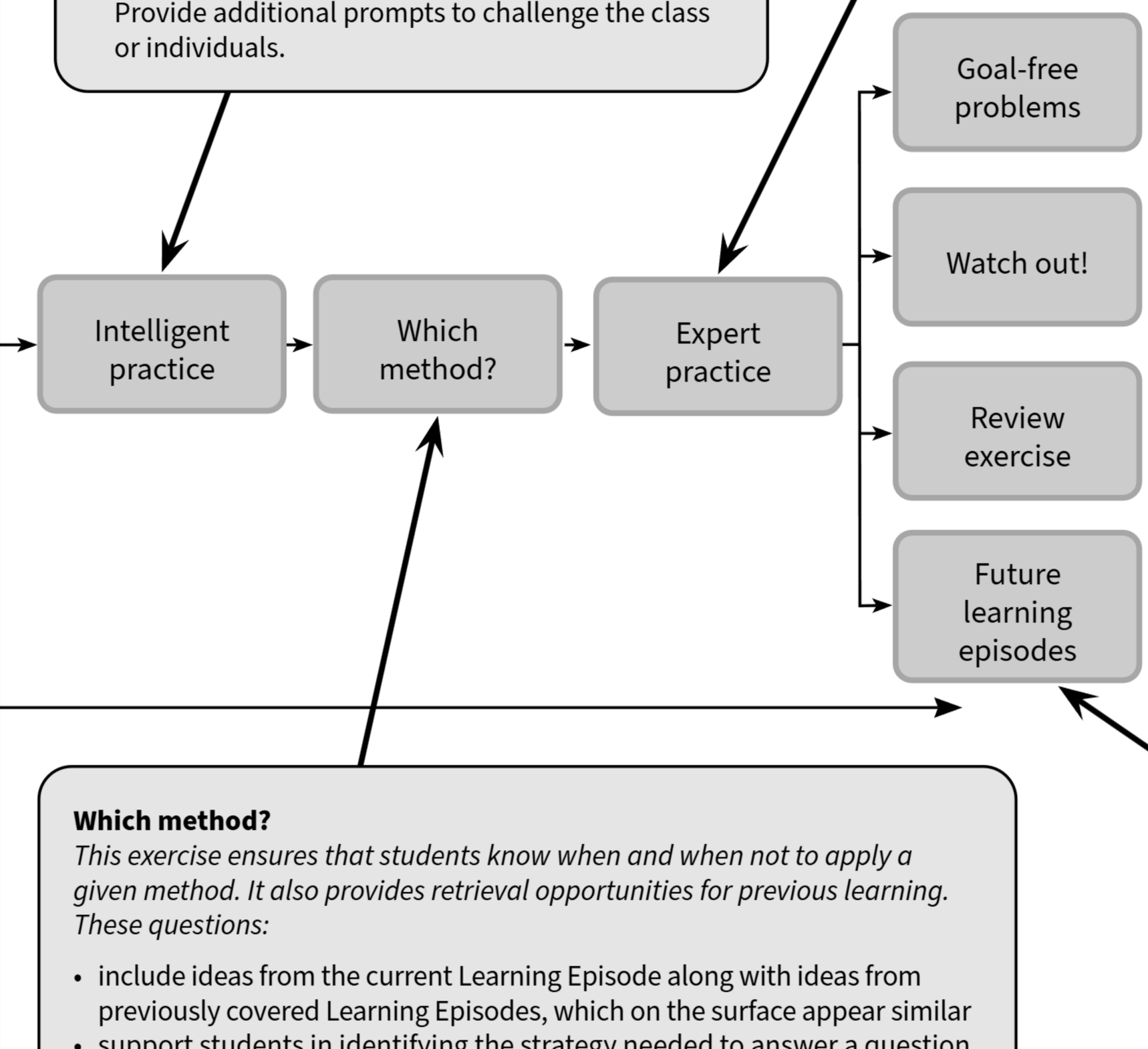
**5 Prompt to delve deeper**
Provide additional prompts to challenge the class or individuals.

**Expert practice**

*These questions are less structured than in the Intelligent practice exercise. They provide practice of a key method, while providing opportunities for students to think more deeply and with greater purpose.*

Five principles:

**1** Students need to experience early success to support motivation.
**2** There must be plenty of opportunities to practise the key procedure.
**3** The practice should feel different to prevent boredom and allow students to see things from a different angle.
**4** Opportunities must exist for students to make connections, solve problems, and think more deeply.
**5** The focus is always on the practice, allowing students to make connections at their own pace.




**Goal-free problems**
Working on these types of problem is more likely to secure the understanding of concepts and connections due to reduced pressure on working memory. Goal-free problem exercises are provided at the end of each chapter in this Teacher's Guide.

**Watch out!**
This exercise introduces students to common misconceptions in the topic areas they have just been studying, at a point when their understanding is secure enough to understand them. This activity also supports students' reasoning skills. They are provided as downloadable worksheets on Kerboodle.

**Review exercise**
These end-of-chapter fluency questions, with mark scheme, encourage students to reflect on their learning and test themselves to see if they have understood the content.

**Which method?**

*This exercise ensures that students know when and when not to apply a given method. It also provides retrieval opportunities for previous learning. These questions:*

- include ideas from the current Learning Episode along with ideas from previously covered Learning Episodes, which on the surface appear similar
- support students in identifying the strategy needed to answer a question
- highlight the importance of reading the question
- allow students to appreciate exactly what it is about a question that requires a given method.

**1** Model how to approach a question and work out what method is needed.
**2** The *Reflect, Expect, Check, Explain* process could be used to support students' thinking here. They can *reflect* on what is familiar about the question and what links they can see to other questions or topics, make an *expectation* about what method is needed, try the question and *check* their answer, and then *reflect* on whether that was the correct method and how they might approach similar questions in future.

# Reflect, Expect, Check, Explain (RECE)

The Reflect, Expect, Check, Explain (RECE) model is a pedagogy that encourages student self-reflection when answering questions. RECE exploits the Self-Explanation Effect, which shows how inviting students to narrate their thought processes while completing tasks can improve understanding and long-term recall. In maths, it is a powerful way to form connections between concepts and help students reflect on how what they already know can help them understand what they do not.

Following the Reflect, Expect, Check, Explain prompts is particularly impactful when students are practising questions, where only one or two elements change between questions. RECE supports students to **reflect** on what they know and on the basis of this make predictions about what they would **expect** to happen. After completing the question, students **check** their answer and then attempt to **explain** why the answer is, or is not, what they expected.

The RECE model can be used in a variety of ways across a Learning Episode and this Teacher's Guide will prompt you where it is especially useful – for example, during direct modelling as part of an EPP-fluency cycle or Intelligent practice. Using RECE promotes more general reflective thinking, which has benefits throughout students' learning journey and ultimately supports them to become more confident mathematicians.

The RECE model has been designed to be simple to use, but like any new approach it is important to model first to students. This ensures that they understand how it works and can try it out themselves with support. Here are some prompts that you can use with students when introducing RECE:

- *First, read the question. What do you notice? Based on previous questions that are similar, what has changed and what is the same?*
- *What would you expect to be the answer to this question? What do you need to do next that is the same and different? Will the answer increase/ decrease? etc.*
- *If the answer surprised you, can you explain why? If it did not surprise you (you answered correctly), how could you explain what you did to someone who does not understand yet? If you did not notice the link between the questions or explain the relationship before, can you do so now?*

# Addressing misconceptions

The Education Endowment Foundation (EEF) defines a misconception as 'an understanding that leads to a systematic pattern of errors'. That is, misconceptions can arise when a generalization has been applied outside of the context in which it is useful.

It is important that teachers have knowledge of possible misconceptions so that they can plan to prevent them before they arise, as well as uncovering and addressing them when they do.

At the start of each chapter in this Teacher's Guide, there is information about the common difficulties and misconceptions that may arise during teaching, with suggestions of how to avoid them. These are also discussed in context during the exposition of each Learning Episode.

Some things to consider when planning to address misconceptions:

- Early in the Learning Episode, focus only on the misconceptions that you **know** students hold.
- Consider how a misconception might have developed. What generalizations has the student misapplied and what counter examples could you use to challenge their belief?
- Plan opportunities for discussion, either as a class or in small groups.
- Use examples, questions, models, and images that allow students to make accurate generalizations and conclusions.
- Compare examples and non-examples of a concept, as well as standard and non-standard representations.

# 1 Place value

## Introduction to chapter

This chapter builds on students' prior learning of place value, and lays the foundations for future mathematical success. Students have already started learning about the place value of digits in integers and decimals. This is extended with much greater and much smaller numbers. They will also expand numbers, including using index form. Students already know about multiplying and dividing by positive powers of 10, so this is embedded while extending the learning to negative powers of 10. Learning is applied through metric unit conversions and working with unit prefixes such as mega- and micro-.

### Core concepts

- Number
- The base-10 numeration system
- Comparison

### What have students already learned?

- Rounding whole numbers
- Tenths arise from dividing into 10 equal parts, hundredths arise from dividing into 100 equal parts, and thousandths arise from dividing into 1000 equal parts
- Using place value to multiply mentally
- Dividing numbers by 10 or 100
- Estimating the position of a whole number on a number line
- Using appropriate standard units of length/height, mass, and volume/capacity

### What will students revisit in this chapter?

- Reading, writing, ordering, and comparing numbers up to 10 000 000, determining the value of each digit
- Each place value is 10 times greater than the place value immediately to the right of it
- Using 0 as a place holder
- Multiplying and dividing numbers by 10, 100, and 1000
- Identifying the value of each digit in decimals
- Recognizing and using thousandths, relating them to tenths, hundredths, and decimal equivalents
- Reading, writing, ordering, and comparing decimals
- Using, reading, writing, and converting between standard units, converting measurements of length, mass, volume, and time, using decimal notation
- Comparing and calculating different measures, including money, using decimal notation

## Getting started

p.2

Direct students' attention to the photo and inset question in the Student Book and encourage a discussion:

- *What place value terms can you name? Can you order these by size?* (e.g. tens, hundreds, thousands, millions, billions)
- *We call these numbers 'powers of 10'. How many digits does each power of 10 have?* (e.g. tens have two digits; hundreds have three digits; thousands have four digits; millions have seven digits; billions have ten digits)
- *What would be an appropriate power of 10 to use for the number of people in our school?* (hundreds/thousands, as appropriate)
- *What would be an appropriate power of 10 to use for the number of people in our town/city?* (thousands/hundred thousands/millions, as appropriate)
- *What would be an appropriate power of 10 to use for the number of people in the world?* (billions)

## Teaching strategy

Students may struggle to represent numbers in expanded form with powers of 10 in index form. Spend time on the powers of 10 and their equivalent values, such as $10^5$ and 100 000 or $10^{-2}$ and 0.01, before using these numbers in the expanded form.

## Common learning misconceptions

The idea of dividing or multiplying by a negative power of 10 (such as $10^{-1}$ (0.1) or $10^{-2}$ (0.01)) can be confusing for some students as it challenges misconceptions that multiplication makes something bigger and division makes something smaller. A Gattegno chart can help here, looking at which calculations jump up or down the chart (see Subsection 1.1.2 for more details).

## Broader context

The way we write our numbers has developed over thousands of years into the base-10, or decimal, system we use today. The symbols we use originated in the Indian subcontinent around 1800 years ago and made their way to Europe via Leonardo of Pisa (also known as Fibonacci) in the thirteenth century.

It is theorized that the base-10 system developed from counting with the ten digits on our hands and, while it is used in most of the world, there are still some cultures who use other systems. It can be interesting to explore in class the base-20 system of the ancient Mayans or the base-60 system of the Babylonians, as this helps students see the benefits of the base-10 system. You should also contrast our system with that of Roman numerals, where writing greater numbers becomes increasingly difficult because of the absence of a place value system.

# 1.1 Place value in integers

Students revisit their learning on positive integer place value from primary school. This reinforces their knowledge of numbers up to millions and extends it to billions and trillions. Their focus is drawn to the value of each digit through expanding the number as a sum of multiples of positive powers of ten. This includes writing these powers in index notation, which will be completely new to students.

| | Learning outcomes | | |
|---|---|---|---|
| **Learning objectives** | **Developing** | **Secure** | **Extending** |
| Understand the place value of digits in integers | Identify the value of digits in numbers up to ten million<br>*E.g. What is the value of the digit 4 in 54 022?* | Identify the value of digits in positive integers and that a digit is 10 times the value of the digit in the next smaller place<br>*E.g. What is the value of the digit 4 in 54 022 000?* | Explain the concept of leading zeros and why they are unnecessary<br>*E.g. Which 0s are unnecessary in the number 0 003 400?* |
| Write integers as words and numerals and use positive powers of 10 in place-value tables | Write positive integers up to ten million in words and numerals where there is no more than one zero in the integer<br>*E.g. Write 68 320 in words. Write forty-three thousand, six hundred and ninety-two in numerals.*<br>Know that $10^2 = 100 =$ one hundred, $10^3 = 1000 =$ one thousand, and recognize these in column headings | Write positive integers up to ten million in words and numerals<br>*E.g. Write four hundred and three thousand and nine in numerals.*<br>Know that $10^4 = 10^3 \times 10$ and so on up to $10^7 = 10^6 \times 10$ and recognize column headings with exponent representations up to $10^7$ | Write positive integers in words and numerals<br>*E.g. Write fifty-four million, ten thousand and eight in numerals.*<br>Recognize column headings in exponent form and write positive integers in expanded form<br>*E.g.*<br>$5300 = 5 \times 1000 + 3 \times 100$<br>$= 5 \times 10^3 + 3 \times 10^2$ |

| Tier 2 vocabulary | Tier 3 vocabulary |
|---|---|
| digit, multiply, numeral | index form, integer, leading zero, place value, power of 10 |

## Classroom resources

**Equipment**

You will need:

- base-10 apparatus

**Models**

- place-value tables
- Gattegno chart

## Digital resources

My self-study quiz, Exercise handout, Extra fluency questions, videos

## 1.1.1 Understanding place value in integers p.4

### Objectives

Students will learn how to:

- understand the place value of digits in integers
- write integers as words and numerals and use positive powers of 10 in place-value tables.

Students have worked with place value in primary school, but only up to millions, so their knowledge of greater numbers may not be reliable. They may have worked with base-10 apparatus to help them conceptualize the relative size of the first four columns (ones, tens, hundreds, and thousands) in integers. It is important to build on students' existing understanding, so continue to use this apparatus if they are familiar with it.

It is important to model place value carefully, so that students understand that the value of digits changes depending on where they are written. Show them the number 555 in a place-value table. Point out that each digit '5' represents a different number. The place-value table is important throughout this chapter to support students' understanding of the columnar nature of our number system. Seeing the columns as fixed and the digits as movable, and recognizing that empty columns are filled with zeros, can help them avoid errors.

Next show the number 21 395 in a place-value table. Show students how the table can be used to write a number in words.

Then show students a place-value table for a number in the millions and demonstrate writing it in words.

**Example-problem pair**

This question draws students' attention to the column where the digit '5' is in and shows that its value is different to its column value. Ask students to try 'Your turn!'.

**Your turn! answer**

80 000 or eighty thousand

The Student Book then moves onto *leading* zeros. It is important to show students which zeros can be ignored in an integer and which cannot. Leading zeros are unnecessary. Necessary zeros, or placeholders, are essential because without them, the value of the number would change.

Make sure that students are familiar with the 'not equal' symbol ≠.

To demonstrate leading zeros, you could ask students to type an integer with leading zeros into the calculator. When they press the equals button, the calculator removes the leading zeros. Give students a series of integers and ask them to examine which zeros are removed and which are not. This will help them identify leading zeros reliably. The examples and non-examples in the Student Book show this.

After students complete the Fluency questions, use these to check students' understanding before moving on.

## 1.1.1 Fluency questions: answers p.5

**1** **a** 70
**b** 40 000
**c** 8 000 000

**2** **a** twenty-three thousand and seventy-six
**b** one million, four hundred and thirty-two thousand, six hundred and seven

**3** **a** 17 820
**b** 4 816 601

**4** **a** 40 000
**b** 100 000 000

**5** **a** 50 000 000
**b** 10 000 000 000

**6** **a** seven hundred and twenty thousand and forty-one
**b** nine million, forty-two thousand and seven
**c** three million and sixty

**7** **a** 85 007
**b** 907 063
**c** 8 064 304

**8** **a** none
**b** 000 040 056
**c** 230 000 000 681
**d** 008 901 320

**9** **a** one hundred and twenty-three million, seven hundred and six thousand, and forty-one
**b** 470 654 301

## 1.1.2 Representing place value in integers using powers of 10 p.6

### Objective

Students will learn how to:

- write integers as words and numerals and use positive powers of 10 in place-value tables.

The new learning in this section is writing the column names using index notation, especially $10^0$. Show students a place-value table with the exponent values added. This notation can be explained by looking at the correspondence between the exponent and the number of zeros in the power of 10. This pattern is shown in the table in the Student Book.

Use the place-value table again to show that the value of each column is ten times greater than the one to its right. Use base-10 apparatus to show this as well. Explain that this relationship continues, even if we have no apparatus to visualize it. Point at a number on a Gattegno chart and ask students to say the number as you point higher up the chart (multiplying by 10) or lower down the chart (dividing by 10). Do this for several numbers to help students make connections of multiplying and dividing by 10, 100, and 1000.

### Example-problem pair 1

This question focuses on the relative magnitude of digits in a number. Rather than multiplying or dividing by powers of 10, it focuses on how multiplying by 10 moves you along a column to the left. It also draws students' attention to the fact that multiplying by 10 three times is equivalent to multiplying by 1000, because $10^3 = 1000$. Ask students to try 'Your turn!'.

**Your turn! 1 answer**

1000 times

The Student Book then looks at numbers in their expanded form and the equivalence between expanded form with and without index notation. When we write a number in expanded form, we explicitly write the number as a sum of multiples of 1, 10, 100, and so on. This representation of numbers will be new to most students.

### Example-problem pair 2

This question asks students to write an integer in expanded form. Part **b** looks at writing a number in its expanded form using exponents. Note that this has two possible correct answers, as they could omit '$0 \times 1000$' due to the coefficient of 0. These expansions can look overwhelming if they are long. Use highlighting to make it clear which part of the expansion matches which digit in the original number. Ask students to complete 'Your turn!'.

**Your turn! 2 answers**

**a** $7 \times 100000 + 8 \times 10000 + 9 \times 1000 + 5 \times 10 + 3 \times 1$

**b** $7 \times 10^5 + 8 \times 10^4 + 9 \times 10^3 + 5 \times 10^1 + 3 \times 10^0$

Ensure that students can confidently use exponents on their calculators. The 'Calculator skills' box in the Student Book is a useful reference here, but be aware that the keys may look different on different models of calculator.

After students complete the Fluency questions, use these to check students' understanding before moving on.

## 1.1.2 Fluency questions: answers p.7

**1** **a** 2
**b** 3
**c** 1

**2** 7

**3** **a** 5
**b** 6

**4** **a** $5 \times 1000 + 2 \times 100 = 5 \times 10^3 + 2 \times 10^2$
**b** $7 \times 10\,000 + 8 \times 1000 + 2 \times 100$
$= 7 \times 10^4 + 8 \times 10^3 + 2 \times 10^2$
**c** $9 \times 10\,000 + 1 \times 1000 + 3 \times 10$
$= 9 \times 10^4 + 1 \times 10^3 + 3 \times 10^1$
**d** $1 \times 100\,000 + 4 \times 10\,000 + 5 \times 1000 + 6 \times 100 + 7$
$= 1 \times 10^5 + 4 \times 10^4 + 5 \times 10^3 + 6 \times 10^2 + 7 \times 10^0$

**5** Six hundred and two million, two hundred and fourteen thousand, and seventy-six

## 1.1 Intelligent practice: answers p.8

Support students to use the Reflect, Expect, Check, Explain model by using prompts: *Have you seen a question like this before? What has changed from the previous question part? How do you think that will affect the answer? Can you explain why you got that answer?*

**1** **a** Three hundred/300
**b** Three hundred/300
**c** Three hundred/300
**d** Three thousand/3000
**e** Thirty thousand/30 000
**f** Three hundred thousand/300 000

**2** **a** 5
**b** 2
**c** 2
**d** 2

**3** **a** 5
**b** 5
**c** 5
**d** 0

**4** **a** 735
**b** 7035
**c** 7035
**d** 70 350

**5** **a** 93 851
**b** 98 315
**c** 90 051
**d** 90 051
**e** 90 015

**6** **a** The zeros are not needed. We consider place value starting in the ones column. If you remove the zeros in the number 00 567, the digits 5, 6, and 7 will remain in the same place so they will have the same value.
**b** Both zeros are needed. If you remove the zeros in the number 56 700, the digits 5, 6, and 7 will move place and so they will change value.

**Question 1** uses the same digits but varies their places. Students should think carefully about which zeros are leading, and therefore unnecessary, and which zeros are placeholders, and therefore necessary.

**Questions 2** and **3** ask students to consider only the ten millions column. Because each number is of a different length, this column is not always in the same place on the page. If students are struggling, use a place-value table to help them.

In each of **Questions 4** and **5**, the same digits are used but their places vary in expanded form, with and without indices. Students have to focus on the power of 10 in each term more than the digit itself. They should be careful in the last two parts of each question, where they need to write zeros in places where this is not made explicit. If students omit these zeros, ask them to compare the place value of each digit in their number with the intended value in the question.

**Question 6** assesses students' reasoning, asking them to explain something that might be obvious to them. This conscious reflection can be difficult for some students, so you may have to prompt them with key words or a sentence stem.

## 1.1 Which method?: answers p.9

**1**

| Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| 1 000 000 | 100 000 | 10 000 | 1000 | 100 | 10 | 1 |
| $10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |

**2** **a** 10
**b** 100
**c** 1000
**d** 3600
**e** 36 000 000

**3** **a** =
**b** ≠
**c** =
**d** =

**4** Answers include:
$2\times1000+3\times100+0\times10+4\times1$
$2\times1000+3\times100+4\times1$
$2\times10^3+3\times10^2+0\times10^1+4\times10^0$
$2\times10^3+3\times10^2+4\times10^0$

**5** There are many answers. Here is one:
700, 1700, 2700, 3700, 4700

**6** Columns are in groups of three, with each group of three going 'one …/ten …/hundred …' where the … is ones/thousands/millions, etc.

**7** 0045, 0405, 0450, 4005, 4050, 4500

**8** Paolo is not correct. The reason is that 80 000 is in the ten thousands and 7000 is in the thousands.

**9** Because $3\times10^4$ is in the ten thousands and $4\times10^3$ is in the thousands

**Question 1** asks students to think forwards and backwards about the equivalencies of the column headings in different formats. This flexibility deepens their understanding of place value.

**Question 2** draws attention to the association of 'jumps' along columns with multiplying by powers of 10. This is covered in more detail in Section 1.3. Students learned about this at primary school, but some may need reminding.

**Question 3** draws attention to the position of × and + in expanded form, as well as not needing to write the multiplications with value 0. Part **d** considers that not all equations have the calculation on the right.

**Question 4** asks students to write all forms without scaffolding and helps them develop their appreciation of equivalence. Ask students what different expansions they have seen so far. Which of these is the newest, and therefore least familiar? This may need reinforcing more often.

**Question 5** is open-ended to assess students' ability to think with fewer constraints. Encourage students to explain the approach they took. Ask: *What other approaches could you use to answer the same question?* This will help them appreciate the underlying structure.

**Question 6** focuses again on the place names and positioning. Encourage students to look for patterns and to describe what they see. This develops their reasoning skills.

**Question 7** uses the same digits in different orders. Encourage students to identify which zeros are necessary. Writing the numbers in a place-value table could help them.

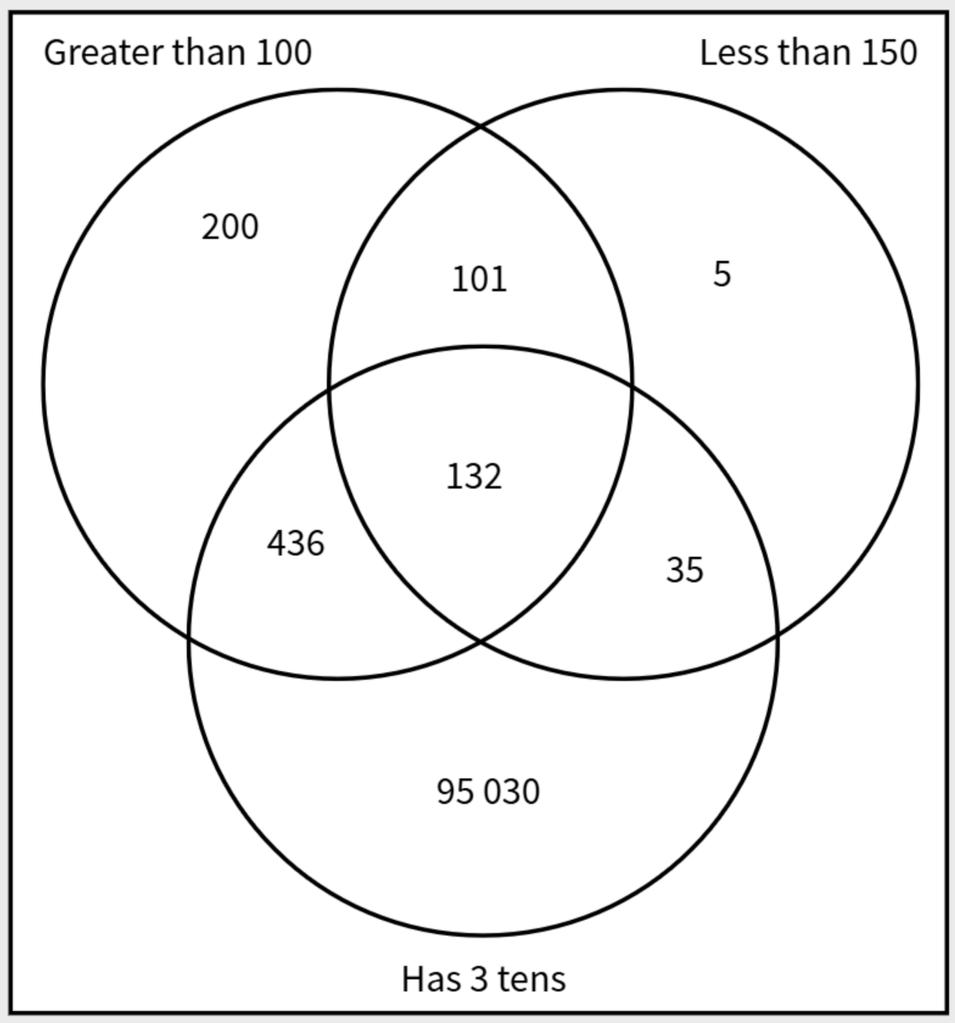
**Questions 8** and **9** check students' understanding that column value is more important than digit value when comparing the relative size of numbers. It also helps students develop skills in reasoning by asking students to explain things that they may have never had to articulate before.

## 1.1 Expert practice: answers



**1** The first number has five digits, the second only has four digits, so the first is greater.

**2** **a** There are many answers. Here is one:




**b** Any number from 130 to 139 can go in the centre section.

**3** **a** 65 410

**b** 01 456 (the question does not specify a five-digit answer)

**c** 65 401

**d** 10 456

**e** 60 145

**4** **a** **i** $41 + 32$ or $42 + 31$

**ii** $13 + 24$ or $14 + 23$

**iii** $43 - 12$

**iv** $34 - 21$

**b** Greatest sum: $432 + 1$ or $431 + 2$; least sum remains the same; greatest difference: $432 - 1$; least difference remains the same

**5** **a** 123 456 789

**b** 987 654 321

**c** 498 765 321

**d** 412 356 789

**e** 698 754 321

**6** **a** 2987

**b** 198 765

**c** 12 345

**Question 1** requires students to consider the number of columns in each integer, not the digits. If students are struggling, ask them to write the numbers in a place-value table, using '?' to represent the unknown digits.

A way into **Question 2** is to start by taking a number with 3 tens and checking which of the lower sections it fits into. Tell students to work in pencil so they can move numbers around easily. Once they start to get a feel for what fits where, they will realize what works in the centre section, allowing them to answer part **b**.

Some students may choose 10 456 for **Question 3** part **b**, but the question does not stipulate a five-digit integer until part **d**. In part **e**, prompt them to consider numbers greater and less than 60 000 to confirm they have found the closest.

**Question 4** relies on students' understanding that the sum of the tens has more impact on the size of the answer than the sum of the ones. There are two possible correct answers for parts **i** and **ii**, but only one for parts **iii** and **iv**. Ask students why this is the case. They learned about commutativity at primary school, so this is a good opportunity to revisit the concept and the vocabulary.

**Questions 5** and **6** are related, except that every digit from 1 to 9 must be used in **Question 5**, whereas there is flexibility to choose in **Question 6**. These questions are similar to **Question 3**, but require a grasp of greater numbers and bring in all the representations we have seen throughout this chapter. If students are struggling, prompt them to start by writing each representation in ordinary, numerical form.

# 1.2 Place value in decimals

Students revisit their learning about place value in positive decimal numbers from primary school, building on what they did with integers in Section 1.1. This reinforces their knowledge of numbers decreasing in size to the magnitude of trillionths, considering the symmetry in the columns between integers and decimals. Their focus is drawn to the value of each digit through expanding the number as a sum of multiples of negative powers of ten. This includes writing these powers in index notation, which will be completely new to students.

| | Learning outcomes | | |
|---|---|---|---|
| **Learning objectives** | **Developing** | **Secure** | **Extending** |
| Understand the place value of digits in decimals | Identify the value of digits in decimals with up to four digits after the decimal point<br>*E.g. What is the value of the digit 5 in the number 0.4582?* | Identify the value of digits in numbers after the decimal point and that a digit is 10 times bigger than the value of the digit in the next smaller place<br>*E.g. What is the value of the digit 5 in the number 9.870 654?* | Explain the concept of trailing zeros and why they are unnecessary<br>*E.g. Which 0s are unnecessary in the number 0.405 00?* |
| Write decimals as words and numerals and use negative powers of 10 in place-value tables | Write decimals with up to three digits after the decimal point in words and numerals<br>*E.g. Write the number 0.458 in words. Write the number four hundred and twenty thousandths in numerals.*<br>Know that $0.1 = \frac{1}{10} = 10^{-1}$ = one tenth, $0.01 = \frac{1}{100} = 10^{-2}$ = one hundredth, and $0.001 = \frac{1}{1000} = 10^{-3}$ = one thousandth, and recognize these in column headings<br>*E.g. 0.53 = 5 tenths and 3 hundredths* | Write decimals with up to six digits after the decimal point in words and numerals<br>*E.g. Write the number 0.0458 in words. Write the number fifty-eight ten thousandths in numerals.*<br>Know that $10^{-2} = 10^{-1} \div 10$, $10^{-3} = 10^{-2} \div 10$, and so on to $10^{-6} = 10^{-5} \div 10$ and recognize column headings with exponent representations down to $10^{-6}$ | Write decimals in words and numerals<br>*E.g. Write the number 1.000 045 8 in words. Write the number one and four hundred and fifty-eight ten-millionths in numerals.*<br>Recognize column headings in exponent form and write decimals in expanded form<br>*E.g.* $0.53 = 5 \times 0.1 + 3 \times 0.01 = 5 \times 10^{-1} + 3 \times 10^{-2}$ |

| | | | |
|---|---|---|---|
| Multiply and divide by positive powers of 10 | Multiply and divide by 10, 100, or 1000 and use a calculator to check the answer<br>*E.g. $48 \div 100$* | Multiply or divide by any positive power of 10 and use a calculator to check the answer<br>*E.g. $48 \times 10\,000$*<br>*$450 \div 100\,000$* | Combine successive operations into a single, equivalent operation, where the operations use positive powers of 10, and use a calculator to check the answer<br>*E.g. $\times 10 \times 100 \div 10\,000$ is equivalent to $\div 10$* |
| Multiply and divide by negative powers of 10 | Multiply or divide by 0.1 and use a calculator to check the answer<br>*E.g. $48 \times 0.1$* | Multiply or divide by any negative power of 10 and use a calculator to check the answer<br>*E.g. $48 \div 0.001$* | Combine successive operations into a single, equivalent operation, where the operations use negative powers of 10, and use a calculator to check the answer<br>*E.g. $\times 10 \times 0.01 \div 100$ is equivalent to $\div 1000$ or $\times 0.001$* |

| Tier 2 vocabulary | Tier 3 vocabulary |
|---|---|
| calculation, decimal, divide, multiply | place value, trailing zero |

## Classroom resources

**Equipment**

You will need:

- base-10 apparatus

**Models**

- place-value tables

## Digital resources

My self-study quiz, Exercise handout, Extra fluency questions, videos

## 1.2.1 Understanding place value in decimals p.12

### Objectives

Students will learn how to:

- understand the place value of digits in decimals
- write decimals as words and numerals and use negative powers of 10 in place-value tables.

Start by reviewing that when a number is split into ten equal sized parts, each part is one-tenth of the whole. Show how this is written as both a fraction and a decimal and how 0.1 is shown in a place-value table, linking this to the column heading — tenths. Repeat with 0.01 and hundredths. Highlight where the decimal point is shown in the place-value table.

The place-value table in the Student Book shows columns up to millionths. Students have worked with place value of decimals to thousandths in primary school, but less so than with integers. They may have worked with base-10 apparatus to help them conceptualize the relative size of ones, tenths, hundredths, and thousandths, where the large cube represents the ones, the flat square tenths, the stick hundredths, and the small cube thousandths. It is important to build on students' existing understanding, so continue to use this apparatus if they are familiar with it.

The place-value table remains of huge importance in helping students conceptualize numbers, but you may find more misconceptions in this section than in Section 1.1. This is partly because splitting a whole into parts is difficult to imagine as the parts get smaller. It becomes all the more important to help students see that as we move to the right through the columns of a number, we are getting ten times less with each jump.

This subsection brings in an extra representation for the column headings (the fractional representation), which can help with the idea of smaller and smaller parts. It can also help with reading the numbers. Some students may read '0.56' as 'zero point fifty-six', because they do not realize that the naming of the columns now works in the opposite direction to the integer columns. Spend time reading numbers out loud with these students. Write decimals on the board and point to different digits, asking them to tell you their value. Continue until all students are confident with it, as they will struggle to proceed further without this confidence.

**Example-problem pair**

This question draws students' attention to the column that the digit '6' is in, but also that its *value* is different to its *column value*. Ask students to complete 'Your turn!' to make sure they are comfortable with the meanings of value and column value.

**Your turn! answer**

Three hundred thousandths

The Student Book then moves on to *trailing zeros*. It is important to show students which zeros can be ignored in a number and which cannot. Trailing zeros are unnecessary. Necessary zeros, or placeholders, are essential because without them the value of the number would change.

Just as with leading zeros in Section 1.1, you can demonstrate trailing zeros with a calculator. Ask students to type a number with trailing zeros into the calculator. When they press the 'equals' button, the calculator removes them. Give students a series of numbers and ask them