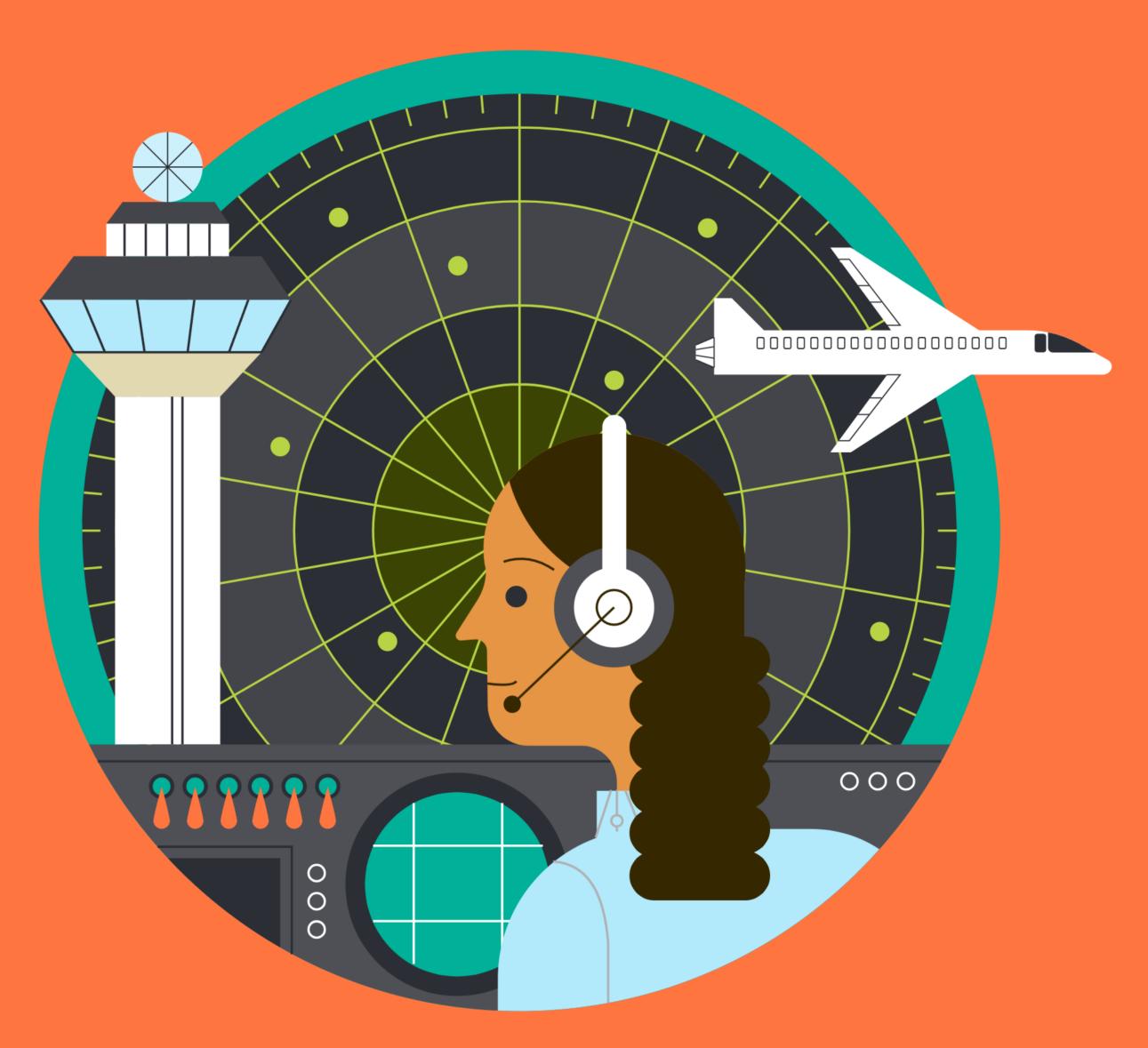


# Maths

Student Book



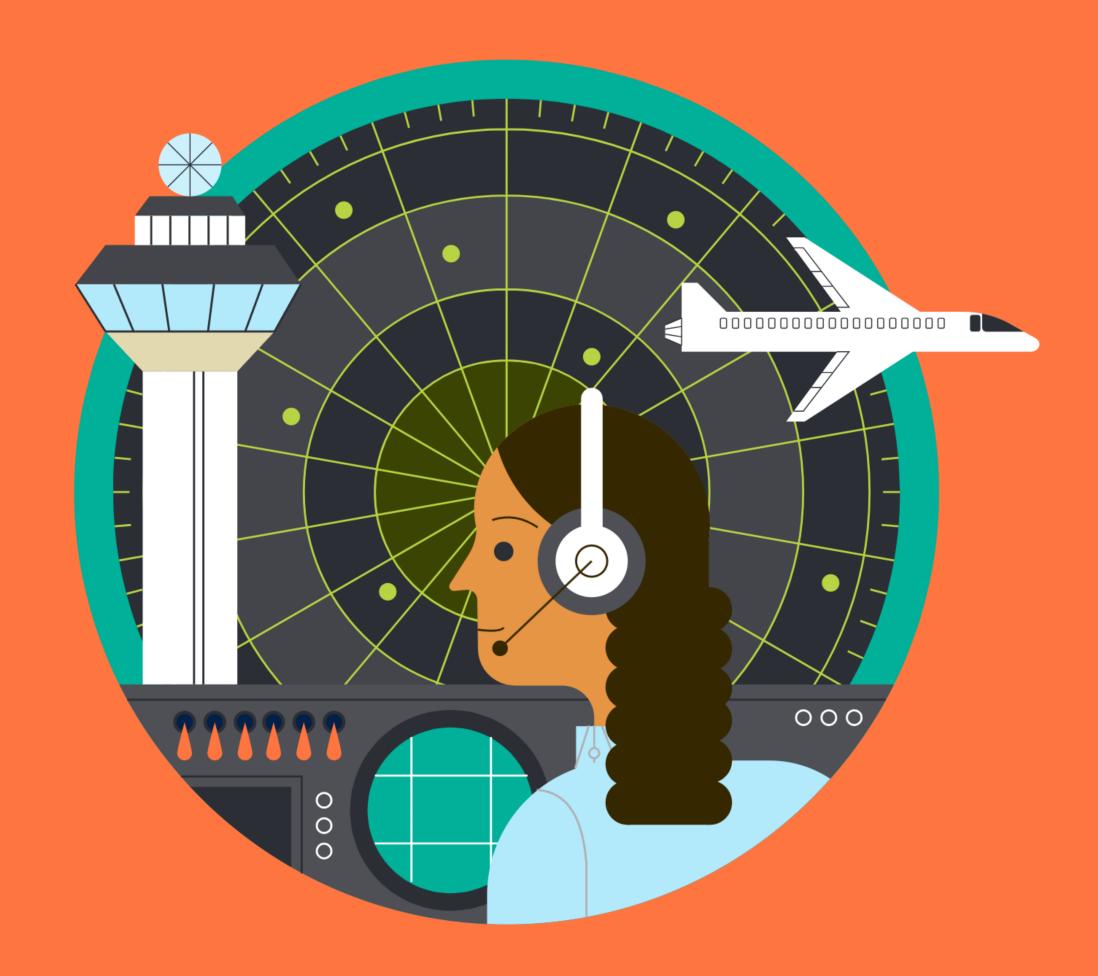
**Lower Secondary** 

OXFORD



# Maths

# Student Book



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OXFORD



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# Contents

How to use this book	iv-v
How to use example-problem pairs	vi
How to use Reflect, Expect, Check, Explain	vii

Chapter 1: Place value	2	Chapter 6: Perimeter and area	214
<b>1.1</b> Place value in integers	4	<b>6.1</b> Quadrilaterals and triangles	216
1.2 Place value in decimals	12	<b>6.2</b> Perimeter	224
1.3 Ordering and comparing numbers	24	<b>6.3</b> Area	232
1.4 Measures	30	What have I learned about perimeter	
What have I learned about place value?	36	and area?	248
Chapter 2: Properties of numbers	38	Chapter 7: Fractions	250
2.1 Multiples	40	7.1 Working with fractions and decimals	252
2.2 Powers and roots	48	7.2 Comparing and ordering fractions	266
2.3 Factors and prime factorization	60	7.3 Adding and subtracting fractions	276
What have I learned about properties		<b>7.4</b> Multiplying and dividing fractions	284
of numbers?	76	What have I learned about fractions?	300
Chapter 3: Arithmetic	78	<b>Chapter 8: Ratio and proportion</b>	302
<b>3.1</b> Adding and subtracting negative integers	80	8.1 Multiplicative relationships	304
<b>3.2</b> Multiplying and dividing negative integers	94	8.2 Representing multiplicative relationships	314
<b>3.3</b> Adding and subtracting decimals	104	8.3 Fractions in context	326
<b>3.4</b> Multiplying and dividing decimals	114	<b>8.4</b> Applying ratios	336
<b>3.5</b> Efficient calculations	126	What have I learned about ratio and	
What have I learned about arithmetic?	140	proportion?	350
Chambau 4. Francesiana and american	4.40	<b>Chapter 9: Transformations</b>	352
Chapter 4: Expressions and equations		9.1 Translations	354
<b>4.1</b> Introduction to algebra	144	9.2 Rotations	364
<b>4.2</b> Formulae and equations	156	9.3 Reflections	374
<b>4.3</b> Simplifying expressions	164	9.4 Scale diagrams	386
<b>4.4</b> Using the distributive law	174	9.5 Enlargements	396
What have I learned about expressions		What have I learned about transformations?	408
and equations?	186		
Chapter 5: Coordinates	188		
<b>5.1</b> Plotting coordinates	190		
<b>5.2</b> Coordinates, formulae, and graphs	200		
What have I learned about coordinates?	212		

Glossary	410
Answers	417

# How to use this book

Each topic begins with a set of learning objectives. These tell you what you will be able to do by the end of the lesson.

#### **Key idea**

The key idea summarizes the main points of each topic in a few sentences.

#### **Key words**

The key words for each topic are highlighted in **bold** in the text. They are also included in order of appearance in this box. You can also find them in the Glossary at the back of your Student Book.

#### Fluency questions

These questions check your understanding of a topic before moving on to the next.



#### Stretch zone

This icon shows you where you will need to think more deeply. It is OK if you find the 'stretch zone' questions difficult. Instead of giving up, keep thinking and trying. You will get there – doing challenging work is the exercise your brain needs!

## **Welcome to your Student Book**

This introduction shows you all the different features *Oxford International Maths* has to support you on your journey through Lower Secondary Maths.

Being a mathematician (someone who studies maths) is great fun. As you work through this Student Book, you will learn how to work mathematically and become confident (or even more confident!) in your maths skills.

Each chapter in this book covers a few topics. With plenty of worked examples and practice questions, you will study these topics for a few weeks to make sure you have time to learn them properly.

#### **Literacy skills**

These boxes tell you more about the history and use of key vocabulary to put the new words you learn in context.

#### **Stretch zone**

These boxes suggest ideas for how to take your learning further and discover something more than what is in the pages of your Student Book.

#### **Calculator skills**

These boxes help you get to know your calculator and use it effectively. Some of the topics in this book would be almost impossible without a calculator. In other topics, your calculator will be useful for doing lots of calculations quickly or for checking answers.

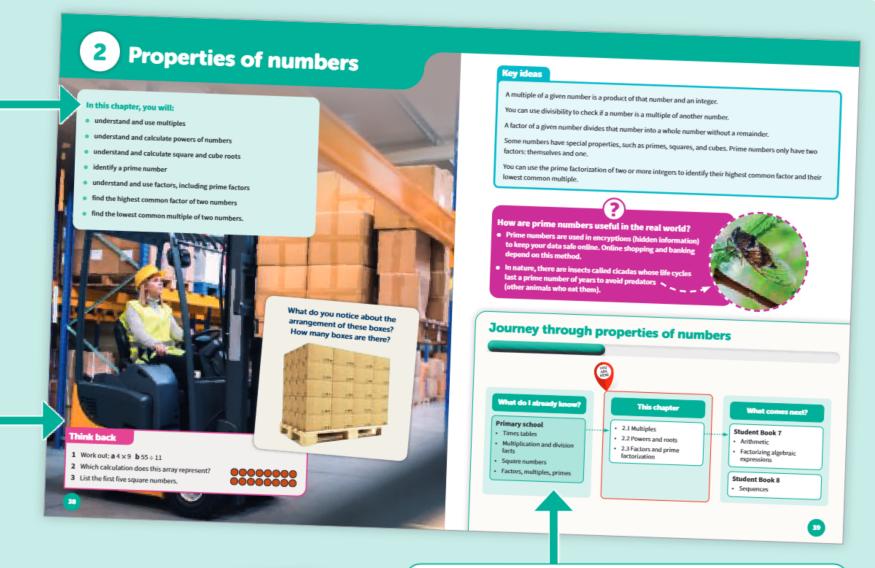
**Do not use your calculator for all your maths**. You should still be confident to carry out calculations, both in your head and by using written methods. Your teacher will tell you when to use a calculator and when they want you to work out a maths problem without one.

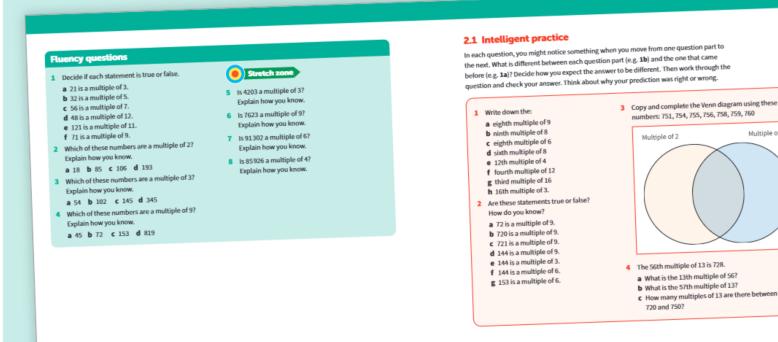
#### **Chapter opener**

Each chapter begins with an introduction. This reminds you what you already know and shows you what is coming up in the chapter.

#### Think back

These quick questions help you recall the maths you already know. To be successful with a new topic in maths, you need to build on your existing knowledge and fill in any gaps before carrying on.





#### **Chapter map**

This map shows clearly what maths you already know, the new topics you will study in this chapter, and the next steps in your maths learning.

#### Become an expert at each topic

There are three different ways to practise maths at the end of every section:

- 1 Intelligent practice
- 2 Which method?
- 3 Expert practice

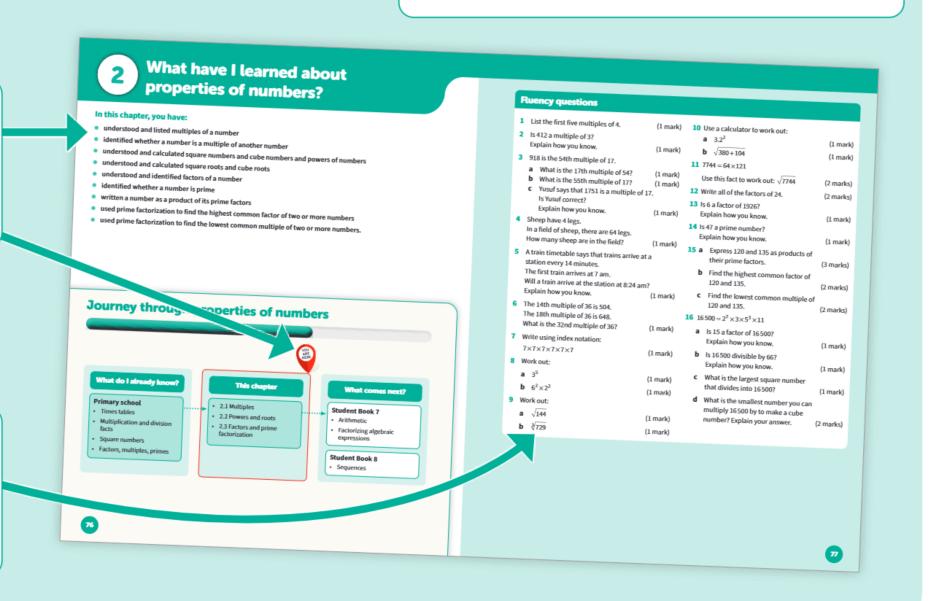
Each exercise works in a particular way to help your brain make connections, remember the topic, and recognize when to use it.

#### **Chapter summary**

This summarizes what you have learned so far and shows your progress through the unit.

#### **Fluency questions**

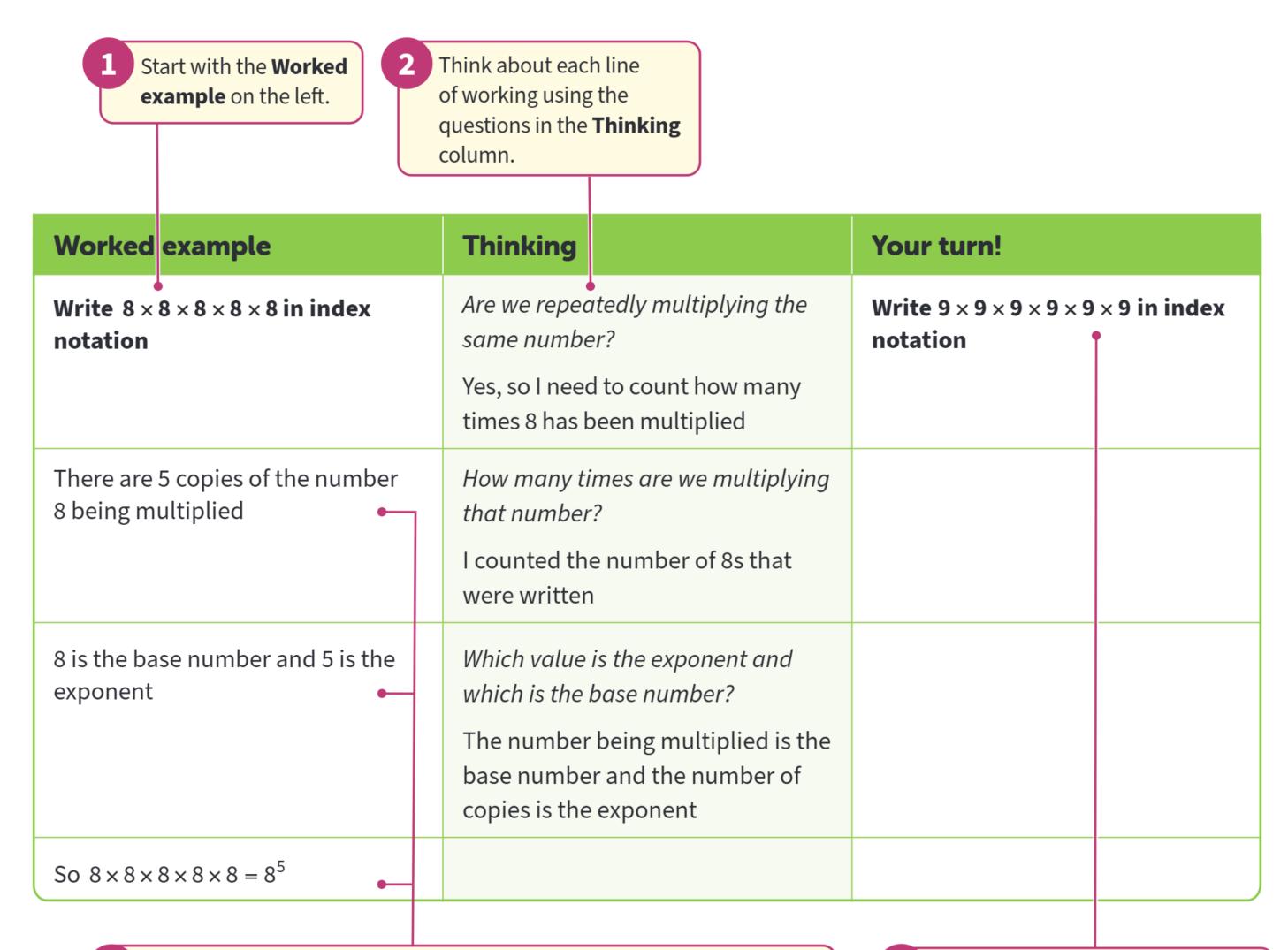
You can use these exam-style questions to test how well you know the topics in the chapter.



# How to use example-problem pairs

**Example-problem pair** (EPP) grids are a special type of worked example that help you understand what you are doing at each step and why.

There are lots of different ways you can use the example-problem pairs. Here is one possible way:



3 Try to predict what the next line of working will be before you look at it.

The questions in the **Thinking** column help you think more generally about the example, so that you understand how to think about a different question.

The **Your turn!** question lets you apply the new idea with some support, so that you can be confident in what you need to do before you move on to the **Fluency** questions.

Once you have thought about the example on the left, move to the Your turn! question on the right.

This question will be very similar to the example you have just studied.

You can use the same thinking ideas to answer this question one step at a time.

# How to use Reflect, Expect, Check, Explain

For the **Intelligent practice** questions, use the Reflect, Expect, Check, Explain (RECE) method. This means you think about the question you are about to do, compare it to the one you have just done, and predict how the answer will be different. This is a great technique for developing your reasoning skills – plus it gives you an opportunity to discuss things with your partner, or as a class, which helps you become more confident talking about maths.

- Reflect: Read the question. What has changed in this question compared to the previous one? What has stayed the same?
- **Expect**: Using your reflection from Step 1 and the answer to the previous question, what do you think the answer will be? Can you explain why you think that?
- Check your expectation by carrying out the usual method to answer the question.
- **Explain**: Was your expectation in Step 2 correct? If the answer surprises you, can you explain why? If the answer is what you expected, how could you explain your reasoning to someone else? If you were not able to make a prediction in Step 2, can you explain the relationship now?

Look at the example below.

#### **Question 2a**

Is 72 a multiple of 9? How do you know?

I recognize 72 from times tables:  $72 = 9 \times 8$ . So, yes, 72 is a multiple of 9.

Or, I could use the divisibility rule for 9.

The digit sum is 7 + 2 = 9 and 9 is a multiple of 9, so 72 must be a multiple of 9.

#### **Question 2b**

Is 720 a multiple of 9? How do you know?

**Reflect:** This question is like 2a because it deals with multiples of 9. The number is just larger.

**Expect:** 720 is 10 × 72. So, 720 must be a multiple of 9 too.

**Check:** Using the divisibility rule for 9, the digit sum 7 + 2 + 0 is 9 and 9 is a multiple of 9.

**Explain:** I was right! 720 is a multiple of 9 because the digit sum is a multiple of 9 and  $72 = 9 \times 8$ ,

so  $720 = 9 \times 80$ .

You can also use this method when you are working through the **Your turn!** question in an example-problem pair.

# 1 Place value

#### In this chapter, you will:

- identify the value of digits in integers and decimals
- write integers and decimals using words and numerals
- multiply and divide by positive and negative powers of 10
- compare and order positive numbers
- recognize good choices of units and convert between them
- order and compare measures.

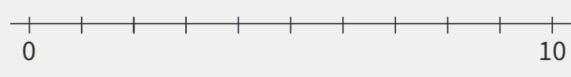
## Think back

**1** Work out:

**a** 800 × 9

**b** 51 ÷ 100

2 Copy this number line and mark the correct place for each of these numbers.



**a** 2

**d** 6.5 **c** 8

**3** For each of these units, state whether they measure length, mass, or volume.

How many snow geese do you

think are in this flock of birds?

**Hundreds? Thousands? Millions?** 

a g

**b** ml

c km

### **Key ideas**

You can use powers of 10 to show the place value of each digit in a number.

Multiplying or dividing a number by 10 changes the place value of each digit.

You can use the symbols <, >, and = to compare two numbers.

You can use your knowledge of place value to order numbers and to convert between units of measure.



#### How have numbers changed in 2000 years?

- 2000 years ago, people used Roman numerals to write numbers.
- Today we use the base-10 system. The base-10 system uses
  the position of digits to tell you the value of each digit. You can
  use this system because of the invention of 0. The number 0 (zero)
  lets us tell the difference between 102, 12 and 120.



# Journey through place value



#### What do I already know?

#### **Primary school**

- Tenths, hundredths, and thousandths
- Reading, writing, ordering, and comparing numbers up to 10 000 000
- Multiplication and division by multiples of 10
- Measurement

#### **This chapter**

- 1.1 Place value in integers
- 1.2 Place value in decimals
- 1.3 Ordering and comparing numbers
- 1.4 Measures

#### What comes next?

#### **Student Book 7**

- Ordering positive and negative numbers
- Comparing and ordering fractions
- Scale diagrams

#### **Student Book 8**

· Estimation and rounding

#### **Student Book 9**

Standard form

# 1.1 Place value in integers

## 1.1.1 Understanding place value in integers

After this topic, you will be able to:

- understand the place value of digits in integers
- write integers as words and numerals and use positive powers of 10 in place-value tables.

#### **Key idea**

The position of a digit in an integer tells you its value.

#### **Key words**

integer, numeral, digit, leading zero

Any positive or negative whole number, including zero, is called an **integer**. A **numeral** is the name for an integer written in **digits**. Digits are the single symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

From the right, the digits in an integer tell you how many ones, tens, hundreds, thousands (and so on) the integer is made up of.

The integer 21 395 is made up of 5 ones, 9 tens, 3 hundreds, 1 thousand, and 2 ten thousands. You can show this in a place-value table:

Ten thousands	Thousands	Hundreds	Tens	Ones
10 000	1000	100	10	1
2	1	3	9	5

The place-value table helps you see how to write a number in words. 21 395 is written twenty-one thousand, three hundred and ninety-five.

The column headings carry on to the left. You will see a pattern in each group of three headings.

	Millions			Millions Thousands			Ones		
Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds Tens		Ones	
100 000 000	10 000 000	1000000	100 000	10 000	1000	100	10	1	

Worked example	Thinking	Your turn!
What is the value of the digit 5 in the integer 510 762?	Which column in the place-value table is the digit in? The 5 is in the hundred thousands column.	What is the value of the digit 8 in the integer 789 053?
$5 \times 100000 = 500000$		

The integer 21 395 starts in the ten thousands column. If you write zeros before the first digit, they will belong to columns to the left of the ten thousands.

00021395 = 21395

These extra zeros before the first digit are called **leading zeros**. If you removed these leading zeros the size of the number would not change, so they are not needed.

In the integer 203 you have 2 hundreds, 0 tens, and 3 ones. You cannot ignore the zero because if you did, the number would read 23 and you would have 2 tens and 3 ones. This is a different number from 203.

203 is not equal to 23. You write this as  $203 \neq 23$ .

In the number 07 203, the first zero is not needed, but the second zero is needed.

#### **Literacy skills**

The word 'digit' comes from the Latin word digitus, meaning a finger or a toe. This is because people counted on their fingers.

#### **Fluency questions**

- 8 940 372 is an integer.Write the value of these digits.
  - **a** 7
- **b** 4
- **c** 8
- 2 Write each of these numbers in words.
  - **a** 23 076 **b** 1 43 2 607
- 3 Write each of these numbers as a numeral.
  - a seventeen thousand, eight hundred and twenty
  - **b** four million, eight hundred and sixteen thousand, six hundred and one
- 4 120 743 905 is an integer.
  Write the value of these digits.
  - **a** 4 **b** 1
- 5 12 057 346 009 is an integer. Write the value of these digits.
  - **a** 5 **b** 1
- 6 Write each of these numbers in words.
  - **a** 720 041 **b** 9 042 007 **c** 3 000 060

- 7 Write each of these numbers as a numeral.
  - a eighty-five thousand and seven
  - **b** nine hundred and seven thousand and sixty-three
  - **c** eight million, sixty-four thousand, three hundred and four

# Stretch zone

- 8 Which zeros are needed in each of these numbers?
  Copy each number and underline the zeros that are needed. Write 'none' if none of the zeros is needed.
  - **a** 0134
  - **b** 000 040 056
  - c 230 000 000 681
  - **d** 008 901 320
- 9 a Write 123 706 041 in words.
  - **b** Write the number four hundred and seventy million, six hundred and fifty-four thousand, three hundred and one using numerals.

# 1.1.2 Representing place value in integers using powers of 10

After this topic, you will be able to:

• write integers as words and numerals and use positive powers of 10 in place-value tables.

You can also write the column headings of a place-value table as **powers of 10** in **index form**.

	Millions			Thousands			Ones		
Hundred millions	Ten millions	Millions	Hundred Ten thousands thousands		Thousands	Hundreds	Tens	Ones	
100 000 000	10 000 000	1000000	100 000	10 000	1000	100	10	1	
10 <sup>8</sup>	10 <sup>7</sup>	10 <sup>6</sup>	10 <sup>5</sup>	10 <sup>4</sup>	10 <sup>3</sup>	10 <sup>2</sup>	10 <sup>1</sup>	10 <sup>0</sup>	

## **Key idea**

You can use powers of 10 to show the place value of each digit in an integer.

Notice the pattern in the powers of 10:

$$10^0 = 1 10^3 = 10 \times 10 \times 10 = 1000$$

$$10^1 = 10$$
  $10^4 = 10 \times 10 \times 10 \times 10 = 10000$ 

$$10^2 = 10 \times 10 = 100$$
  $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000$ 

### **Key words**

power of 10, index form, multiply

You can see that a digit becomes 10 times greater in value each time it moves one place to the left.

Worked example	Thinking	Your turn!
In 264 368 how many times greater is the first 6 than the second 6?	What do we <b>multiply</b> by to move from one column to the next?  To move left, I multiply by 10.  How many columns do we move from one digit to the other?  There are three places to move between the sixes.	In 5 164 172 how many times greater is the first 1 than the second 1?
$10 \times 10 \times 10 = 1000$ times greater		



The column headings carry on to the left, using higher powers of 10.
Research what the next few columns are.

You can write a number in expanded form using powers of 10 to show the place value of each digit:

$$21395 = 2 \times 10000 + 1 \times 1000 + 3 \times 100 + 9 \times 10 + 5 \times 1$$

You can also write this using powers of 10 in index form as:

$$21395 = 2 \times 10^4 + 1 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 5 \times 10^0$$

#### **Calculator skills**

To enter 10<sup>4</sup> on a calculator, press these keys:









The power key may look like .

Worked example	Thinking	Your turn!
Write the integer 510 762 in expanded form using powers of 10 in a) numerals and b) index form.	What is each digit multiplied by in the number?  The digits are multiplied by 100 000, 10 000, 1000, 100, 10, and 1 in that order.	Write the integer 789 053 in expanded form using powers of 10 in a) numerals and b) index form.
<b>a</b> $510762 =$ $5 \times 100000 + 1 \times 10000 +$ $0 \times 1000 + 7 \times 100 +$ $6 \times 10 + 2 \times 1$	What powers of 10 in index form represent each of the columns?  100 000 is 10 <sup>6</sup> .  10 000 is 10 <sup>4</sup> , and so on.	
<b>b</b> $510762 =$ $5 \times 10^5 + 1 \times 10^4 +$ $0 \times 10^3 + 7 \times 10^2 +$ $6 \times 10^1 + 2 \times 10^0$		

### **Fluency questions**

Here is a number written in a place-value table.

Thousands	Hundreds	Tens	Ones
7	4	2	1

Fill in the numbers in these sentences.

- **a** The value of the digit 4 in the hundreds column is  $4 \times 10^{\square}$
- **b** The value of the digit 7 in the thousands column is  $7 \times 10^{\square}$
- **c** The value of the digit 2 in the tens column is  $2\times10^{\square}$
- 2 Here is a number: 12 057 346 009 Which digit is in the place that has a value 10 times more than the place of the digit 3?
- Here is a number written in a place-value table.

Fill in the numbers in these sentences.

- **a** The value of the digit 1 in the hundred thousands column is  $1 \times 10^{\square}$
- **b** The value of the digit 6 in the ten millions column is  $6 \times 10 \times 10^{\square}$
- 4 Write each of these numbers in expanded form using powers of 10 in numerals and index form. For example:

$$6700 = 6 \times 1000 + 7 \times 100 = 6 \times 10^3 + 7 \times 10^2$$

- **b** 78 200 **a** 5200
- **c** 91 030
- **d** 145 607



Write the number shown in the place-value table in words.

	10 <sup>8</sup>	10 <sup>7</sup>	10 <sup>6</sup>	10 <sup>5</sup>	10 <sup>4</sup>	10 <sup>3</sup>	10 <sup>2</sup>	10 <sup>1</sup>	10 <sup>0</sup>
ı	6	0	2	2	1	4	0	7	6

Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
6	2	1	7	3	0	8	9

### 1.1 Intelligent practice

In each question, you might notice something when you move from one question part to the next. What is different between each question part (e.g. **1b**) and the one that came before (e.g. **1a**)? Decide how you expect the answer to be different. Then work through the question and check your answer. Think about why your prediction was right or wrong.

- 1 What is the value of the digit 3 in each integer?
  - **a** 315
  - **b** 0315
  - **c** 00315
  - **d** 3150
  - **e** 31500
  - **f** 301050
- 2 What is the ten millions digit in each integer?
  - **a** 52 342 456
  - **b** 523 424 567
  - c 678 523 424 567
  - **d** 6783523424567
- **3** What is the ten millions digit in each integer?
  - **a** 456 000 000
  - **b** 056 000 000
  - **c** 00 456 000 000
  - **d** 45 600 000 000
- 4 Here are some integers written in expanded form. Write them in ordinary form.
  - **a**  $7 \times 100 + 3 \times 10 + 5 \times 1$
  - **b**  $7 \times 1000 + 0 \times 100 + 3 \times 10 + 5 \times 1$
  - **c**  $7 \times 1000 + 3 \times 10 + 5 \times 1$
  - **d**  $7 \times 10000 + 3 \times 100 + 5 \times 10$
- 5 Here are some integers written in expanded form. Write them in ordinary form.
  - **a**  $9 \times 10^4 + 3 \times 10^3 + 8 \times 10^2 + 5 \times 10^1 + 1 \times 10^0$
  - **b**  $9 \times 10^4 + 8 \times 10^3 + 3 \times 10^2 + 1 \times 10^1 + 5 \times 10^0$
  - $9 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 1 \times 10^0$
  - **d**  $9 \times 10^4 + 5 \times 10^1 + 1 \times 10^0$
  - **e**  $9 \times 10^4 + 1 \times 10^1 + 5 \times 10^0$
- 6 Which zeros in these numbers are needed and which are not needed? Explain why.
  - **a** 00 567
  - **b** 56 700

#### 1.1 Which method?

In these questions, you will need to think carefully about which methods to apply. For some questions, you might need to use skills from earlier chapters.

1 This table shows the names of the place-value columns in words, numerals, and powers of 10. Copy the table and fill in the gaps.

		Hundreds	Tens	Ones
	10 000	100	10	1
10 <sup>6</sup>		10 <sup>2</sup>	10 <sup>1</sup>	

- 2 Copy these and fill in the boxes to make each of the calculations correct.
  - **a**  $36 \times \square = 360$
  - **b**  $36 \times \square = 3600$
  - **c**  $36 \times \square = 36000$
  - **d**  $\square \times 100\,000 = 360\,000\,000$
  - **e**  $\square \times 10 = 360\,000\,000$
- **3** Write the correct symbol,  $= \text{ or } \neq$ , for each box.
  - **a**  $5068 \square 5 \times 1000 + 0 \times 100 + 6 \times 10 + 8 \times 1$
  - **b**  $5068 \square 5 + 1000 \times 0 + 100 \times 6 + 10 \times 8 + 1$
  - **c**  $5068 \square 5 \times 10^3 + 6 \times 10^1 + 8 \times 10^0$
  - **d**  $5 \times 1000 + 6 \times 10 + 8 \times 1 \square 5068$
- 4 Write the number 2304 in expanded form with powers of 10 in three different ways (using numerals or index form).
- 5 Write down five numbers, each greater than the one before, that contain 7 hundreds.
- 6 In the place-value table, the column headings can be put in groups of three:

		Millions			Thousands			Ones		
		Hundred	Ten	One	Hundred	Ten	One	Hundreds	Tens	Ones
١	•••	millions	millions	millions	thousands	thousands	thousands	Tidilalcas	10113	Offics

Describe the pattern you see in the column headings.

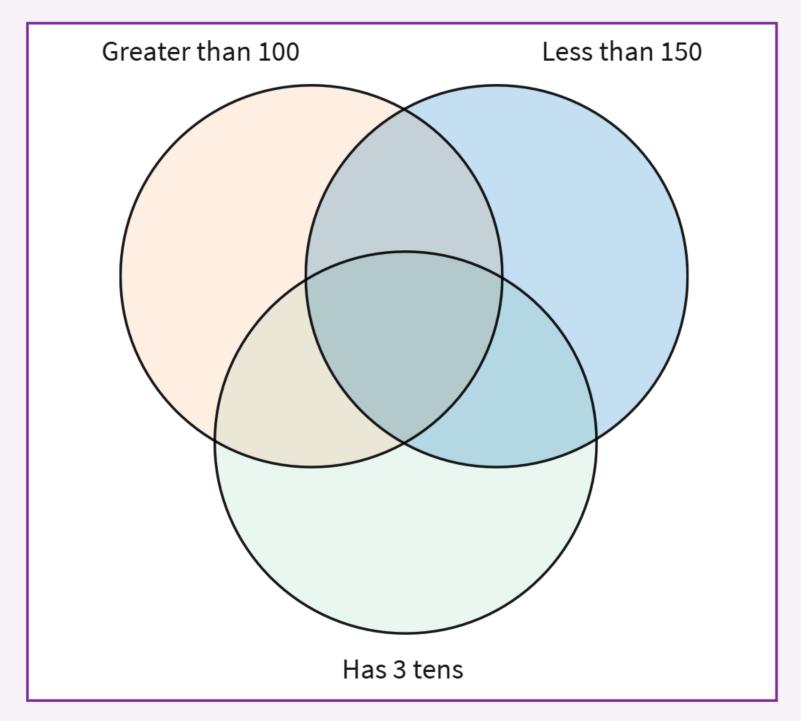
- 7 Write these numbers in ascending order. 0450, 4500, 4050, 4005, 0405, 0045
- Stretch zone
- 8 Paolo says, 'The number 80 000 is greater than the number 7000 just because 8 is greater than 7.' Is Paolo correct? Explain your reasoning.
- **9** Explain how you know that the number  $3 \times 10^4$  is greater than the number  $4 \times 10^3$ .

#### 1.1 Expert practice

There may be more than one way to look at these questions.

Once you have answered a question one way, can you think of another way?

- 1 Here are two integers with some of their digits hidden. Which number is greater? How do you know?
  - 4 19 9 1
- **a** Copy and complete the Venn diagram by filling each section with an integer. If a section cannot be filled, explain why.



- **b** Is there more than one number that can go in the centre section? Explain how you know.
- **3** Here are five digits:

5 1 0 4 6

Use these digits to make:

- **a** the greatest possible integer
- **b** the least possible integer
- **c** the greatest possible odd integer
- **d** the least possible five-digit, even integer
- **e** the integer closest to 60 000.

4	<b>a</b> By putting the digits 1, 2, 3, and 4 into the boxes once only, make:
	i the greatest possible sum
	ii the least possible sum
	iii the greatest possible difference
	iv the least possible difference.
	<b>b</b> How do your answers to part <b>a</b> change if you can move the + or – symbol anywhere between the
	four digits?
5	Using all the digits 1 to 9 only once, make:
	a the integer closest to 100 000 000
	<b>b</b> the integer closest to 1 000 000 000
	c the integer closest to five hundred million
	$f d$ the least integer greater than $4 \times 100000000$
	<b>e</b> the greatest integer less than $7 \times 10^8$ .
6	Using any of the digits from 1 to 9 no more than once, make:
	a the integer closest to 3000
	<b>b</b> the greatest integer less than 2×100 000
	c the least integer greater than 10 <sup>4</sup> .

# 1.2 Place value in decimals

## 1.2.1 Understanding place value in decimals

After this topic, you will be able to:

- understand the place value of digits in decimals
- write decimals as words and numerals and use negative powers of 10 in place-value tables.

#### **Key idea**

The position of a digit in a decimal tells you its value.

If you split the number 1 into 10 equal-sized parts, each part is one tenth of the whole. You write this as  $\frac{1}{10}$  in fraction notation or 0.1 in decimal notation. In a **decimal**, the whole numbers go to the left of the decimal point, and the parts of a whole number go to the right of the decimal point. The parts of a whole are shown to the right of the ones in a place-value table:

Ones	Tenths	Hundredths	Thousandths	Ten- thousandths	Hundred- thousandths	Millionths
1	$\frac{1}{10}$	1 100	1 1000	1 10 000	$\frac{1}{100000}$	$\frac{1}{1000000}$
1	0.1	0.01	0.001	0.0001	0.00001	0.000001
0	2	1	7	4	8	3

### **Literacy skills**

When reading numbers, people may say 'nought' instead of 'zero', so you will often hear 'nought point ...'.

The column names for parts of a whole have the same patterns as for integers. You can write the column headings as fractions or decimals.

The place-value table shows the number 0.217 483. The number 0.217 483 is read 'zero point two one seven four eight three'. It is made up of 2 tenths, 1 hundredth, 7 thousandths, 4 ten-thousandths, 8 hundred-thousandths, and 3 millionths.  $0.217483 = 0 \times 1 + 2 \times 0.1 + 1 \times 0.01 + 7 \times 0.001 + 4 \times 0.0001 + 8 \times 0.00001 + 3 \times 0.0000001$ 

Worked example	Thinking	Your turn!
What is the value of the digit 6 in the number 5.107 62?	Which column in the place-value table is the digit in? The 6 is in the ten-thousandths column.	What is the value of the digit 3 in the number 7.890 53?
$6 \times 0.0001 = 0.0006$ , or 6 ten-thousandths		

The number 0.213 95 ends in the hundred-thousandths column. If you write zeros after the last digit, they will belong to columns to the right of the hundred-thousandths.

0.21395000 = 0.21395

These extra zeros are called **trailing zeros**. If you removed these trailing zeros the size of the number would not change, so they are not needed.

In the number 1.203 you have 1 one, 2 tenths, 0 hundredths, and 3 thousandths. You cannot ignore the zero because if you did, the number would read 1.23 and would have 1 one, 2 tenths, and 3 hundredths. This is a different number from 1.203.

1.203 is not equal to 1.23. You write this as  $1.203 \neq 1.23$ .

For the number 0.7230, the first zero is needed but the second is not needed.

### **Key words**

decimal, trailing zero

### **Fluency questions**

- Here is a number: 0.5423Write the value of these digits.
  - **a** 5
  - **b** 2
- Write each of these numbers in words.
  - **a** 0.12
  - **b** 0.045
- 3 Write each of these numbers using numerals.
  - a seven tenths
  - **b** seventeen hundredths
- 4 Here is a number: 0.123 984 Write the value of these digits.
  - **a** 9
  - **b** 8
- 5 Here is a number: 15.942 307 006 Write the value of these digits.
  - **a** 7
  - **b** 6
- 6 Write each of these numbers in words.
  - **a** 0.2456
  - **b** 0.0364
  - **c** 0.941 245
  - **d** 0.875 016

- 7 Write each of these numbers using numerals.
  - a five hundred and seven thousandths
  - **b** eighty-five ten-thousandths
  - c one thousand, four hundred and seventeen ten-thousandths
  - **d** eight hundred and seventy-one thousand, six hundred and fifty-seven millionths

# Stretch zone

- 8 Which zeros are needed in each of these numbers? Copy each number and underline the zeros that are needed. Write 'none' if none of the zeros is needed.
  - **a** 0.40
  - **b** 0.024 500
  - **c** 0.145 000 030
  - **d** 0.005 000 000 100
  - **e** 0.000 000 123 547
- Write the number nineteen and sixty-five millionths using numerals.

# 1.2.2 Representing place value in decimals using powers of 10

After this topic, you will be able to:

• write decimals as words and numerals and use negative powers of 10 in place-value tables.

### **Key idea**

You can use powers of 10 to show the place value of each digit in a decimal.

You can also write the column headings of a place-value table as powers of 10 in index form.

Ones	Tenths	Hundredths	Thousandths	Ten- thousandths	Hundred- thousandths	Millionths
1	$\frac{1}{10}$	1 100	1 1000	1 10 000	$\frac{1}{100000}$	$\frac{1}{1000000}$
1	0.1	0.01	0.001	0.0001	0.00001	0.000001
10 <sup>0</sup>	$10^{-1}$	$10^{-2}$	10 <sup>-3</sup>	10 <sup>-4</sup>	10 <sup>-5</sup>	10 <sup>-6</sup>
0	2	1	7	4	8	3

#### **Calculator skills**

To enter 10<sup>-5</sup> on a calculator, press these keys:



The power key may look like ( ).

Notice the pattern in the powers of 10:

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^2 = 10 \times 10 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

As you move from each column to the one on its right, the value of the column is divided by 10. The negative powers of 10 show that you are looking at parts of a whole.

Worked example	Thinking	Your turn!
In 0.056 2306 how many times greater is the first 6 than the second 6?	What do we multiply by to move from one column to the next?  To move left, I multiply by 10.  How many columns do we move from one digit to the other?  There are four places to move between the sixes.	In 1.000 01 how many times bigger is the first 1 than the second 1?
$10 \times 10 \times 10 \times 10 = 10000$ times greater.		

The place-value table helps you see how to write a number in expanded form.

$$0.217483 = 0 \times 1 + 2 \times 0.1 + 1 \times 0.01 + 7 \times 0.001 + 4 \times 0.0001 + 8 \times 0.00001 + 3 \times 0.0000001$$

You can use powers of 10 in index form to write a decimal in expanded form.

$$0.217483 = 0 \times 10^{0} + 2 \times 10^{-1} + 1 \times 10^{-2} + 7 \times 10^{-3} + 4 \times 10^{-4} + 8 \times 10^{-5} + 3 \times 10^{-6}$$

Worked example	Thinking	Your turn!
Write the number 5.107 62 in expanded form using a) numerals and b) powers of 10.	What is each digit multiplied by in the number? The digits are multiplied by 1, 0.1, 0.01, 0.001, 0.0001, and 0.00001 in that order.	Write the number 7.890 53 in expanded form using a) numerals and b) powers of 10.
<b>a</b> $5.10762 = 5 \times 1 + 1 \times 0.1 + 0 \times 0.01 + 7 \times 0.001 + 6 \times 0.0001 + 2 \times 0.00001$	What powers of 10 represent each of the columns?  1 is $10^0$ , 0.1 is $10^{-1}$ , 0.01 is $10^{-2}$ , and so on.	
<b>b</b> $5.10762 = 5 \times 10^{0} + 1 \times 10^{-1} + 0 \times 10^{-2} + 7 \times 10^{-3} + 6 \times 10^{-4} + 2 \times 10^{-5}$		

## **Fluency questions**

1 Here is a number written in a place-value table.

Ones	Tenths	Hundredths	Thousandths	
0	6	7	3	

Fill in the numbers in these sentences.

- **a** The value of the digit 7 in the hundredths column is  $7 \times 10^{\square}$
- **b** The value of the digit 6 in the tenths column is  $6 \times 10^{\square}$
- c The value of the digit 3 in the thousandths column is 3×10□
- 2 Here is a number: 15.942 307 006 Which digit is in the place that has a value 10 times less than the place of the digit 4?
- 3 Here is a number written in a place-value table.

Fill in the numbers in these sentences.

- **a** The value of the digit 8 in the thousandths column is  $8 \times 10^{\square}$
- **b** The value of the digit 6 in the millionths column is  $6 \times 10 \times 10^{\square}$



4 Write each of these numbers in expanded form using powers of 10 in numerals and index form. For example:

$$0.67 = 6 \times 0.1 + 7 \times 0.01 = 6 \times 10^{-1} + 7 \times 10^{-2}$$

- **a** 0.56 **b** 0.871 **c** 0.7021 **d** 0.0306
- 5 Write the number shown in the place-value table in words.

10 <sup>0</sup>	$10^{-1}$	10 <sup>-2</sup>	$10^{-3}$	$10^{-4}$	10 <sup>-5</sup>	10 <sup>-6</sup>	$10^{-7}$
3	0	7	5	2	1	0	3

Ones Tenths Hundred		Hundredths	Thousandths	Ten- thousandths	Hundred- thousandths	Millionths
0	4	5	8	1	0	6

# 1.2.3 Multiplying and dividing by positive powers of 10

After this topic, you will be able to:

• multiply and divide by positive powers of 10.

#### **Key idea**

Multiplying a number by 10 moves each digit one place value to the left.

Dividing a number by 10 moves each digit one place value to the right. The value of each column in a number is ten times greater than the column to the right. This means that when you multiply a number by 10, all the digits get ten times greater and move one column to the left.

 $45 \times 10 = 450$ 

Since 100 is  $10 \times 10$  multiplying by 100 moves the digits two places to the left. In a similar way, multiplying by 1000 moves them three places, multiplying by 10000 moves them four places, and so on.

Worked example	Thinking	Your turn!
Work out 6.7×10 000	Which way do the digits move when we multiply by 10, 100, 1000,?  They move to the left.  How many places do the digits need to move? $10000 = 10^4$ , so the digits move four places.	Work out 0.23×1000
$6.7 \times 10000 = 67000$		

Dividing by 10 makes the digits in a number move one place to the right.

 $4.5 \div 10 = 0.45$ 

In the same way, dividing by 100 moves the digits two places to the right, dividing by 1000 moves three places, and so on.

Worked example	Thinking	Your turn!
Work out 4300 ÷ 100 000	Which way do the digits move when we divide by 10, 100, 1000,?  They move to the right.  How many places do the digits need to move?  100 000 = 10 <sup>5</sup> , so the digits move five places.	Work out 45.8 ÷ 10 000
4300 ÷ 100 000 = 0.043		

# **Fluency questions**

- 1 Work out:
  - **a** 34×10
  - **b** 57×100
  - **c** 91÷10
  - **d** 69751÷1000
- 2 Work out:
  - **a** 2.3×10000
  - **b**  $36 \times 100000$
  - **c** 6.31×1000000000
  - **d** 452 ÷ 10 000
  - **e** 694125÷1000000
  - **f** 947 254.63 ÷ 1 000 000
- **3** Work out:
  - **a**  $6.12 \times 10^3$
  - **b**  $3.41 \times 10^5$
  - **c**  $0.82 \times 10^6$
  - **d**  $12.73 \times 10^7$
  - **e**  $6790 \div 10^3$
  - **f**  $123540 \div 10^5$



- 4 Work out:
  - **a**  $6.1 \times 10 \times 1000 \times 100$
  - **b**  $45.102 \times 10 \times 100 \times 1000 \times 100$
  - **c**  $675124 \div 100 \div 1000 \div 100$
  - **d**  $641312 \div 1000000 \times 1000 \times 10000000 \div 10000$
- 5 Work out:
  - **a**  $5.4 \times 10^2 \times 10^3$
  - **b**  $16.2 \times 10^4 \times 10^5$
  - **c**  $8.7 \times 10^6 \div 10^4$
  - **d**  $91.5 \times 10^8 \div 10^7 \times 10^2$

# 1.2.4 Multiplying and dividing by negative powers of 10.

After this topic, you will be able to:

• multiply and divide by negative powers of 10.

## **Key idea**

Multiplying a number by  $\frac{1}{10}$  moves each digit one place value to the right.

Dividing a number by  $\frac{1}{10}$  moves each digit one place value to the left.

#### **Key word**

calculation

You can also multiply and divide by numbers such as 0.1, 0.01, 0.001, ...

Since 0.1 is  $\frac{1}{10}$ , multiplying by 0.1 is the same as dividing by 10.

$$45 \times 0.1 = 45 \div 10 = 4.5$$

This means multiplying by 0.01 is the same as dividing by 100, multiplying by 0.001 is the same as dividing by 1000, and so on.

Notice the pattern in these **calculations**:

$$6 \times 1000 = 6000$$

$$6 \times 100 = 600$$

$$6 \times 10 = 60$$

$$6 \times 1 = 6$$

$$6 \times 0.1 = 0.6$$

$$6 \times 0.01 = 0.06$$

$$6 \times 0.001 = 0.006$$

Worked example	Thinking	Your turn!
Work out 905×0.1	Which way do the digits move when we multiply by 0.1, 0.01, 0.001,?  Multiplying by 0.1 is the same as dividing by 10, so they move to the right.	Work out 1.05×0.01
	How many places do the digits need to move?  10 = 10 <sup>1</sup> , so the digits move one place.	
$905 \times 0.1 = 90.5$		

Multiplying by 0.1 is the same as dividing by 10. In a similar way, dividing by 0.1 is the same as multiplying by 10.

 $6 \div 1000 = 0.006$ 

 $6 \div 100 = 0.06$ 

 $6 \div 10 = 0.6$ 

 $6 \div 1 = 6$ 

 $6 \div 0.1 = 60$ 

 $6 \div 0.01 = 600$ 

 $6 \div 0.001 = 6000$ 

Worked example	Thinking	Your turn!
Work out 0.012 ÷ 0.001	Which way do the digits move when we divide by 0.1, 0.01, 0.001,?  Dividing by 0.001 is the same as multiplying by 1000, so they move to the left.  How many places do the digits need to move?  1000 = 10 <sup>3</sup> , so the digits move three places.	Work out 0.0054 ÷ 0.000 01
12		

# **Fluency questions**

- 1 Work out:
  - a  $52\times0.1$
- **b**  $7.3 \times 0.1$
- 2 Work out:
  - **a**  $46 \times 0.001$
  - **b**  $590 \times 0.0001$
  - **c** 92700×0.000001

- **c**  $85 \div 0.1$
- **d**  $645 \div 0.01$
- **e** 8430 ÷ 0.001
- **f**  $0.6987 \div 0.00001$
- **3** Work out:
  - **a**  $81 \times 10^{-3}$
  - **b**  $761 \times 10^{-5}$

- **d**  $6.1 \div 0.1$

- **c**  $45 \div 10^{-2}$
- **d**  $34.79 \div 10^{-4}$

# Stretch zone

- 4 Work out:
  - **a**  $5.31 \times 1000 \div 0.1 \times 0.001 \div 10$
  - **b**  $732 \div 0.000001 \times 1000000 \div 0.1 \div 100000$
  - **c**  $8.2146 \times 0.000001 \times 1000 \div 0.01 \div 0.001$
- 5 Work out:
  - **a**  $67.2 \times 10^2 \div 10^{-3} \times 10^{-4}$
  - **b**  $19.2 \div 10^{-5} \times 10^{-4} \div 10^{-2}$
  - **c**  $0.754 \times 10^{-6} \div 10^{-5} \div 10^{-4} \times 10^{-7}$

## 1.2 Intelligent practice

In each question, you might notice something when you move from one question part to the next. What is different between each question part (e.g. 1b) and the one that came before (e.g. 1a)? Decide how you expect the answer to be different. Then work through the question and check your answer. Think about why your prediction was right or wrong.

**d** 0.3150

**e** 31.500

**f** 0.00301050

What is the value of the digit 3 in each number?

**a** 3.15

**b** 0.315

**c** 0.0315

What is the ten millionths digit in each number?

**a** 0.52342456

**b** 0.523424567

**c** 0.678523424567

**d** 0.678520024567

What is the ten millionths digit in each number?

**a** 0.00000456

**b** 0.45600000

**c** 0.00456000000

**d** 0.00000045600

Here are some numbers written in expanded form. Write them in ordinary form.

a  $7 \times 0.1 + 3 \times 0.01 + 5 \times 0.001$ 

**b**  $7 \times 0.1 + 0 \times 0.01 + 3 \times 0.001 + 5 \times 0.0001$ 

 $\mathbf{c}$  7×0.1+3×0.001+5×0.0001

**d**  $7 \times 1 + 3 \times 0.01 + 5 \times 0.0001$ 

Here are some numbers written in expanded form. Write them in ordinary form.

**a**  $9 \times 10^{-1} + 3 \times 10^{-2} + 8 \times 10^{-3} + 5 \times 10^{-4} + 1 \times 10^{-5}$ 

**b**  $9 \times 10^{-1} + 8 \times 10^{-2} + 3 \times 10^{-3} + 1 \times 10^{-4} + 5 \times 10^{-5}$ 

 $9 \times 10^{-1} + 0 \times 10^{-2} + 0 \times 10^{-3} + 5 \times 10^{-4} + 1 \times 10^{-5}$ 

**d**  $9 \times 10^{-1} + 5 \times 10^{-4} + 1 \times 10^{-5}$ 

**e**  $9 \times 10^{-1} + 1 \times 10^{-4} + 5 \times 10^{-5}$ 

Which zeros in these numbers are needed and which are not needed? Explain why.

**a** 5.6700

**b** 0.0567

#### 1.2 Which method?

In these questions, you will need to think carefully about which methods to apply. For some questions, you might need to use skills from earlier chapters.

1 This table shows the names of the place-value columns in words, numerals, fractions, and powers of 10. Copy the table and fill in all the gaps.

Ones	Tenths	Hundredths			
1	0.1	0.01			0.00001
	1		_1_		
	10		1000		
	10 <sup>-1</sup>			10 <sup>-4</sup>	

2 Copy these and fill in the boxes to make the calculations correct.

a	36×	□ =	= 360

**b** 
$$36 \div \square = 360$$

**c** 
$$360 \times \square = 36$$

**d** 
$$3600 \times \square = 36$$

**e**  $36\,000 \div \square = 36$ 

**f** 
$$36\,000 \times \square = 36$$

**g** 
$$\square \times 0.00001 = 3.6$$

**h** 
$$\square \div 0.00001 = 3.6$$

**3** Copy these and put the correct symbol,  $= \text{ or } \neq$ , into each box.

**a** 
$$5.068 \square 5 \times 1 + 0 \times 0.1 + 6 \times 0.01 + 8 \times 0.01$$

**c** 
$$5.068 \square 5 \times 10^{0} + 6 \times 10^{-2} + 8 \times 10^{-3}$$

**b** 
$$5.068 \square 5 + 1 \times 0 + 0.1 \times 6 + 0.01 \times 8 + 0.01$$

**d** 
$$5 \times 1 + 6 \times \frac{1}{100} + 8 \times \frac{1}{1000} \square 5.068$$

- 4 Write the number 23.045 in expanded form with powers of 10 in three different ways (using numerals or index form).
- 5 Write down five numbers, each less than the one before, which contain 7 hundredths.
- 6 If I multiply by 10, then again by 10, then again by 10, it is the same as multiplying by 1000, because  $10 \times 10 \times 10 = 1000$ .

Write down a single multiplication that is the same as each of these.

**b** 
$$\times 10 \times 10 \times 1000$$

$$c \times 100 \times 100 \times 100$$

$$\mathbf{d} \times 0.1 \times 0.1$$

**e** 
$$\times 0.01 \times 0.01$$

**f** 
$$\times 0.001 \times 0.1 \times 0.1$$

$$\mathbf{g} \times 10 \times 0.01 \times 100$$

**h** 
$$\times 1000 \times 0.001$$

7 If I divide by 10, then again by 10, then again by 10, it is the same as dividing by 1000, so  $\div 10 \div 10 \div 10$  is equivalent to  $\div 1000$  because  $10 \times 10 \times 10 = 1000$ .

Write down a single division that is the same as each of these.

**a** 
$$\div 100 \div 10$$

**b** 
$$\div 10 \div 10 \div 10000$$

**c** 
$$\div 100 \div 100 \div 100$$

**d** 
$$\div 0.1 \div 0.1$$

$$e \div 0.01 \div 0.01$$

$$\mathbf{f} \div 0.1 \div 0.01 \div 0.001$$

$$\mathbf{g} \div 1000 \div 0.001$$

**h** 
$$\div 100 \div 0.1 \div 1000$$

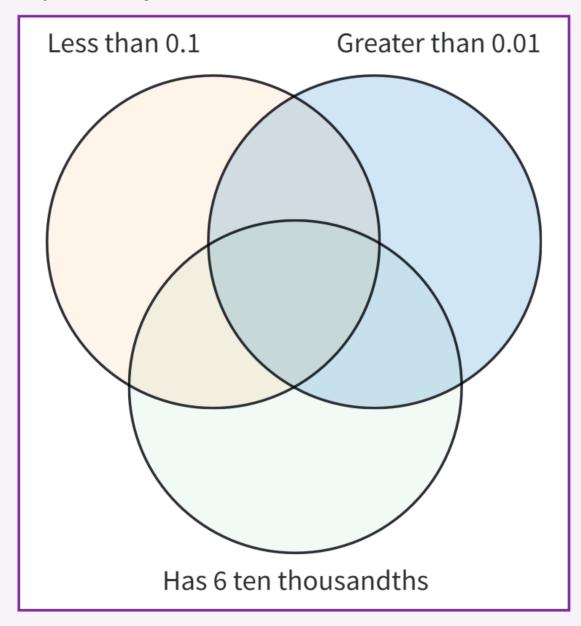
# Stretch zone

- **8** Layla says, 'The number 0.00007 is less than the number 0.0008 because 7 is less than 8.' Is Layla correct? Explain your reasoning.
- **9** Explain how you know that the number  $4 \times 10^{-3}$  is greater than the number  $3 \times 10^{-4}$ .

## 1.2 Expert practice

There may be more than one way to look at these questions. Once you have answered a question one way, can you think of another way?

1 a Copy and complete the Venn diagram with a number that fits each section. If a section cannot be filled, explain why.



**b** Is there more than one number that can go in the centre section? Explain how you know.

**2** Here are five digits:

7 9 0 1 8

Use all of these digits, and a decimal point if necessary, to make:

- a the least possible number
- **b** the number closest to 1
- c the number closest to 80
- **d** the greatest number less than 10
- **e** the least number greater than 7.
- 3 Here are two numbers with some of their digits hidden. Which number is greater? How do you know?

0.62

0.7411